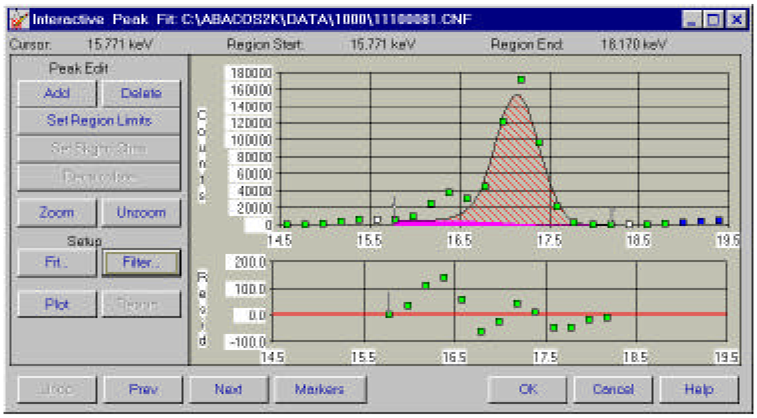
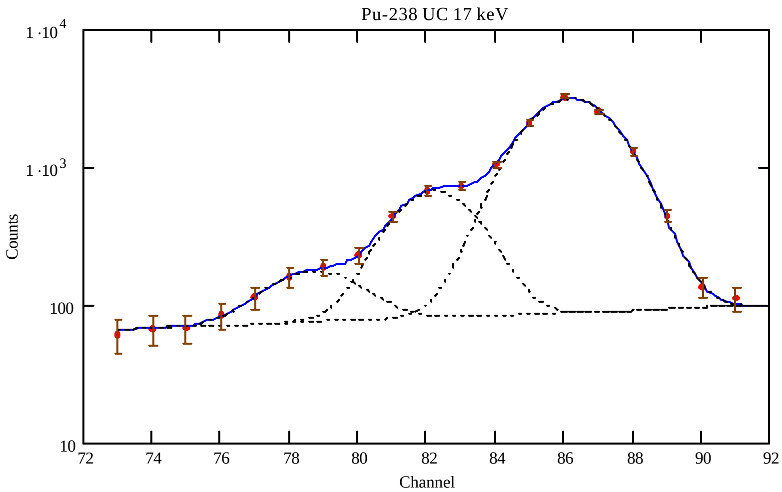
Hand Verification of ABACOS 2K Results Using Mathcad

Thomas R. La Bone

2002 DOE Lung Conference
Augusta, GA
May 7, 2002





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ABSTRACT

ABACOS 2K on a chest counter is a rather complicated computer program and it can be difficult to independently verify that it is arriving at the "correct" answer. As part of our DOELAP accreditation activities we verified the results reported by ABACOS 2K using the simple relationship

$$A_{unknown} = A_{s\tan dard} \cdot \left(\frac{C_{unknown}}{C_{S\tan dard}} \right)$$

where

$A_{unknown}$ = activity in the unknown lung set
$A_{Standard}$ = activity in the calibration lung set
$C_{unknown}$ = net counts in the photopeak in the unknown spectrum
$C_{Standard}$ = net counts in the photopeak in the standard spectrum

Calculating the activity in an unknown lung set in this way gives an answer that is completely independent of ABACOS. This activity can be compared to the activity reported by ABACOS to verify the end result of the calculations performed by ABACOS.

Activities were calculated using both the 17 keV x-ray multiplet and the 43 keV gamma. The only technical challenge here is calculating the area of the photopeak, especially the 17 keV x ray. A non-linear least squares fitting routine in the computer program Mathcad 2001i was used to deconvolute the peaks and calculate the areas. Errors in the peak areas were calculated using a Monte Carlo technique. The results of our study along with the Mathcad worksheets will be presented during this presentation. The worksheets will be made available for those interested in the calculations.

Hand Verification of ABACOS 2k Results Using Mathcad

ABACOS 2K on a chest counter is a rather complicated computer program and it can be difficult to independently verify that it is arriving at the "correct" answer. As part of our DOELAP accreditation activities we verified the results reported by ABACOS 2K using the simple relationship

$$A_{unknown} = A_{Standard} \cdot \left(\frac{C_{unknown}}{C_{Standard}} \right)$$

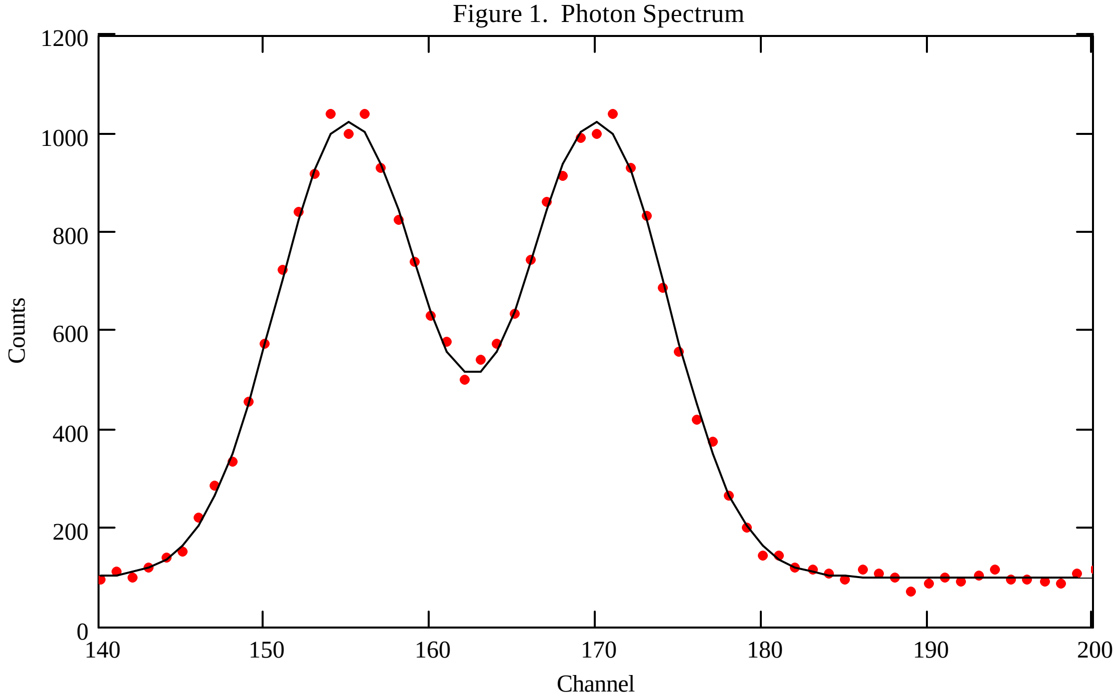
Calculating the activity in an unknown lung set in this way gives an answer that is completely independent of ABACOS. This activity can be compared to the activity reported by ABACOS to verify the end result of the calculations performed by ABACOS.

- **Gaussian Probability Density Function**
- **Non-linear Least Squares Fitting of Peaks**
- **Analysis of UC Pu-238 Lung Set - 43 keV**
- **Analysis of UC Pu-238 Lung Set - 17 keV**
- **Pu-238 Activity in DOELAP Lung Set**

Normal (Gaussian) PDF and Photon Peaks

X-ray and gamma-ray spectra typically consist of a background continuum with one or more Gaussian-shaped peaks on top of the continuum. Multiple peaks are often not completely resolved, as shown in Figure 1 below.

Figure 1. Photon Spectrum



A standard normal (Gaussian) probability density function (PDF) is given by the function f(x,μ,σ) and has a mean of 0 and a standard deviation of 1.

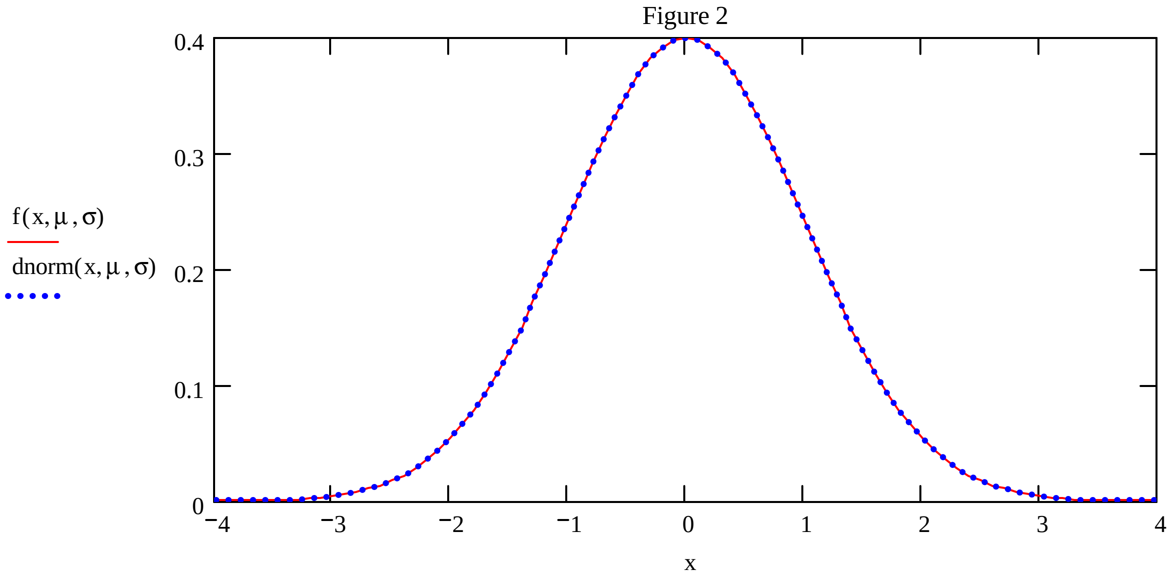
$$\sigma = 1$$

$$\mu = 0$$

$$f(x,\mu,\sigma) = \frac{\exp\left[-\frac{(x-\mu)^2}{2\cdot\sigma^2}\right]}{\sigma\cdot(2\cdot\pi)^{\frac{1}{2}}}$$

A plot of the normal PDF creates the familiar bell curve. Mathcad has a built-in function called **dnorm** that also generates the normal PDF.

$$x = \mu - 4\cdot\sigma, (\mu - 4\cdot\sigma) + 0.1 .. \mu + 4\cdot\sigma$$



Like all PDF, the integral of $f(x,\mu,\sigma)$ over its applicable range is equal to 1.

$$\int_{-\infty}^{\infty} f(x,\mu,\sigma)\,dx = 1$$

This integral is the area of the peak. Thus, the expectation (predicted) counts C_{exp} in the i^{th} channel are given by

$$C_{exp_i} = C_{bkg_i} + A_1\cdot f(i,\mu_1,\sigma_1) + A_2\cdot f(i,\mu_2,\sigma_2)$$

where C_{bkg} is the counts in the background continuum, which often takes the form of a linear, step, or exponential function. The parameters A, μ, and σ of the the peaks and the parameters of the background continuum function may be estimated from a weighted least-squares fit that minimizes the sum of squares (or χ^2) function.

$$SS = \sum_{i=1}^{N} \frac{\left(C_{exp_i} - C_{obs_i}\right)^2}{C_{obs_i}}$$

Note that because μ and σ appear in the exponent of the normal PDF, the SS function is non-linear and there is no analytical solution. An iterative, non-linear least-squares fitting routine in Mathcad is used to solve for the unknown parameters.

Full Width of a Peak at Half-Maximum Height (FWHM)

$$fwhm = \frac{f(0,\mu,\sigma)}{2} = f(X,\mu,\sigma) \text{ solve}, X \rightarrow \begin{pmatrix} 2^{\frac{1}{2}} \cdot \ln(2)^{\frac{1}{2}} \\ -2^{\frac{1}{2}} \cdot \ln(2)^{\frac{1}{2}} \end{pmatrix}$$

$$fwhm = \begin{pmatrix} 1.177 \\ -1.177 \end{pmatrix}$$

$$fwhm_0 - fwhm_1 = 2.355$$

Thus, the FWHM is equivalent to 2.355σ. The full width at tenth-max (FWTM) is calculated in a similar fashion to give 4.292σ .

$$fwtm = \frac{f(0,\mu,\sigma)}{10} = f(X,\mu,\sigma) \text{ solve}, X \rightarrow \begin{pmatrix} 2^{\frac{1}{2}} \cdot \ln(10)^{\frac{1}{2}} \\ -2^{\frac{1}{2}} \cdot \ln(10)^{\frac{1}{2}} \end{pmatrix}$$

$$fwtm = \begin{pmatrix} 2.146 \\ -2.146 \end{pmatrix}$$

$$fwtm_0 - fwtm_1 = 4.292$$

Nonlinear Least Squares Fit to a Peak

The area (A) and width (σ) of a peak will be determined in this worksheet to illustrate how a nonlinear least squares fit works.

$ORIGIN \equiv 1$

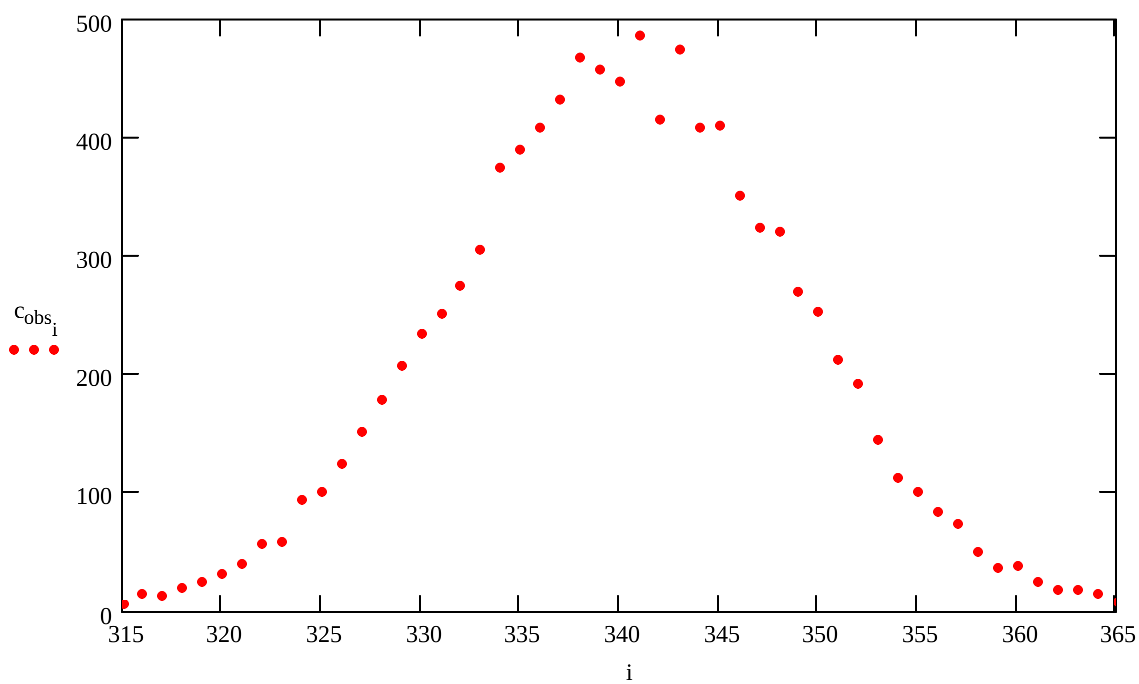
$\begin{pmatrix} Channel \\ c_{obs} \end{pmatrix} :=$

1	0
2	0
3	0
4	0
5	0
6	0

$N := rows(Channel)$

$i := 315..365$

This is what the spectrum looks like.



Initial guesses for the peak area (A), centroid channel (μ), and standard deviation (σ).

$A := 10 \cdot 10^3$

$\mu := 340$

$\sigma := 6$

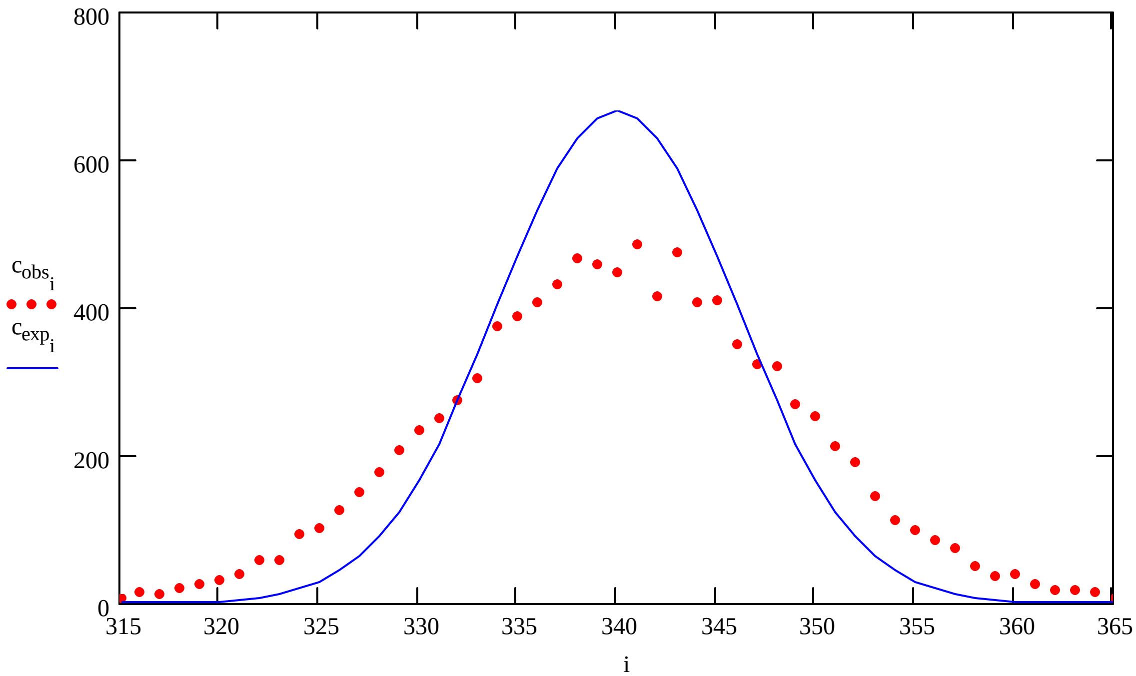
$$f(x,\mu,\sigma) := \frac{\exp\left[-\frac{(x-\mu)^2}{2\cdot\sigma^2}\right]}{\sigma\cdot(2\cdot\pi)^{\frac{1}{2}}}$$

Note that only two parameters (A and σ) will be determined (μ is fixed).

$$C(ch, A, \sigma) := A \cdot \mathrm{dnorm}(ch, \mu, \sigma)$$

View spectrum that results from initial guesses.

$$c_{exp_i} := C(i, A, \sigma)$$



This is the sum of squares function that will be minimized.

$$SS(A,\sigma) := \sum_{i=315}^{365} \frac{\left(c_{obs_i} - C(i,A,\sigma)\right)^2}{c_{obs_i}}$$

Perform a nonlinear weighted least squares fit of the model to the data.

Given

$$SS(A,\sigma) = 0$$

$$\begin{pmatrix} A \\ \sigma \end{pmatrix} := \text{Minerr}(A,\sigma)$$

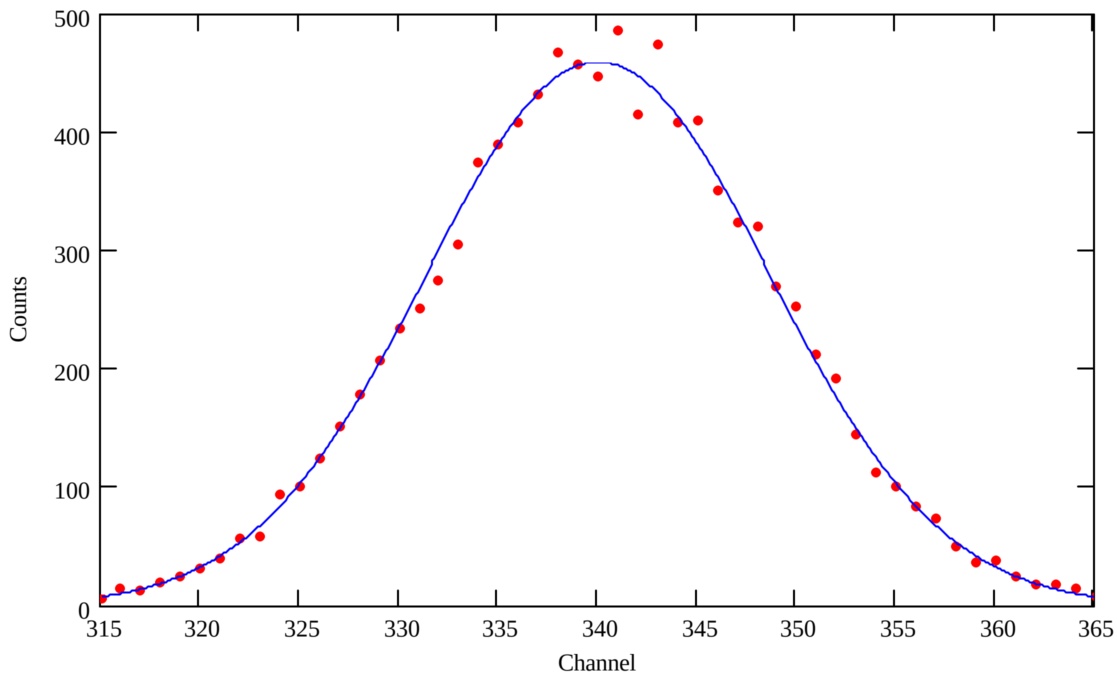
Weighted least squares fit estimates of parameters.

$A = 9.991 \times 10^3$ $\mu = 340$ $\sigma = 8.68$

Generate the spectrum.

$j := 3150..3650$

$$c_{exp_j} := C\left(\frac{j}{10}, A, \sigma\right)$$



Because we are only fitting two parameters, we can examine the sum of squares surface.

$\text{increments} \equiv 50$

$A_{min} \equiv 9000 \qquad \sigma_{min} \equiv 7.5$

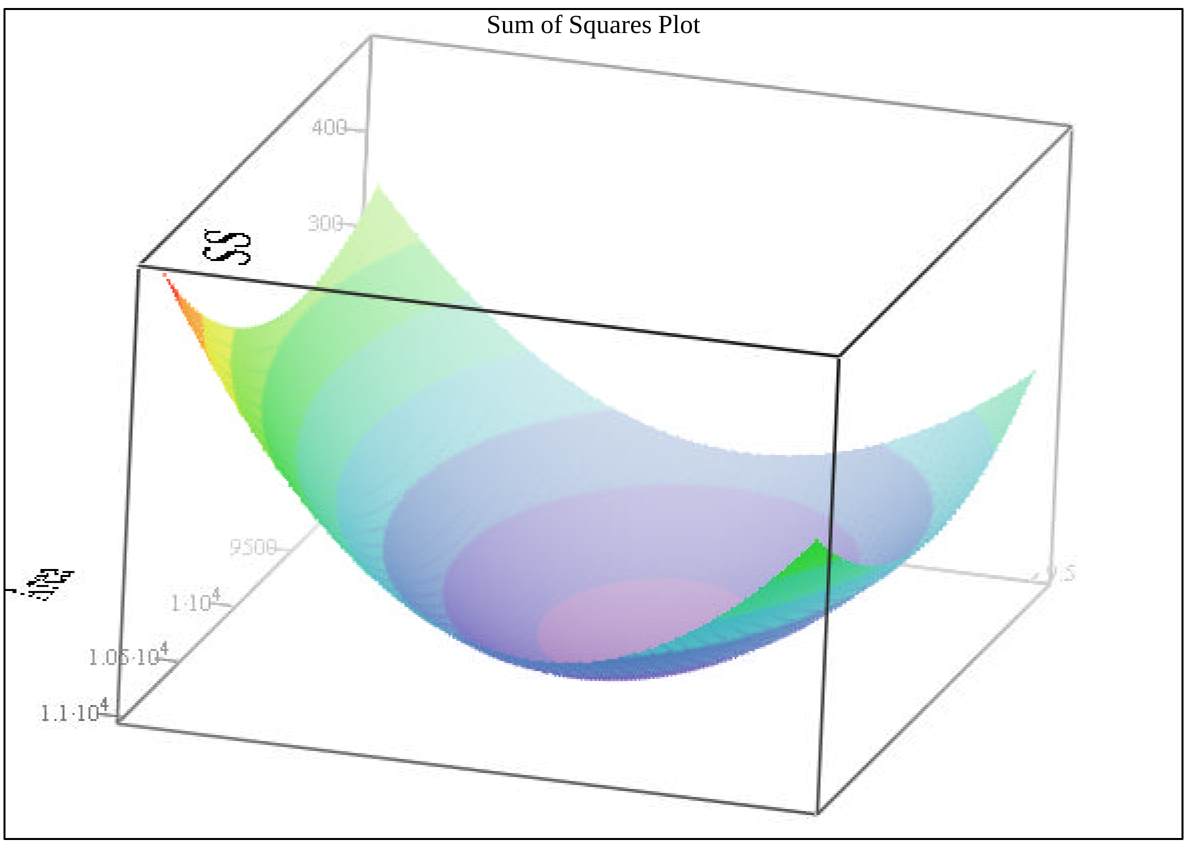
$A_{max} \equiv 11000 \qquad \sigma_{max} \equiv 9.5$

$i := 1 .. \text{increments}$

$j := 1 .. \text{increments}$

Generate x,y,z coordinates for sum of squares surface.

$x_{i,j} := A_{min} + i \cdot \dfrac{A_{max} - A_{min}}{\text{increments}} \qquad y_{i,j} := \sigma_{min} + j \cdot \dfrac{\sigma_{max} - \sigma_{min}}{\text{increments}} \qquad z_{i,j} := SS\left(x_{i,j}, y_{i,j}\right)$



(x, y, z)

SRS savannah river site

Chest Counter

Pu-238 UC Lungs - 43 keV

A 5400 second count of the University of Cincinnati lung set on the bed counting system. The lungs contain 1278.2 nCi of Pu-238 on 9/7/99. The 43 keV gamma ray will be analyzed in this worksheet.

$$\mathrm{ORIGIN} \equiv 1$$

Functions to convert data from ABACOS format to conventional format.

$$\mathrm{Col}(x) := \left| \begin{array}{l} \mathrm{Col} \leftarrow \mathrm{mod}(x,8) \\ \mathrm{Col} \leftarrow 8 \ \text{ if } \ \mathrm{Col} = 0 \end{array} \right.$$

$$\mathrm{Row}(x) := \left| \begin{array}{l} \mathrm{Row} \leftarrow \mathrm{trunc}\left(\frac{x}{8}\right) + 1 \\ \mathrm{Row} \leftarrow \mathrm{trunc}\left(\frac{x}{8}\right) \ \text{ if } \ \mathrm{Col}(x) = 8 \end{array} \right.$$

Counts per channel from Ge detector in ABACOS format. Counts in $\beta_{1,1}$ are from channel 1 and counts from $\beta_{2,1}$ are from channel 9 and so on.

$\beta :=$

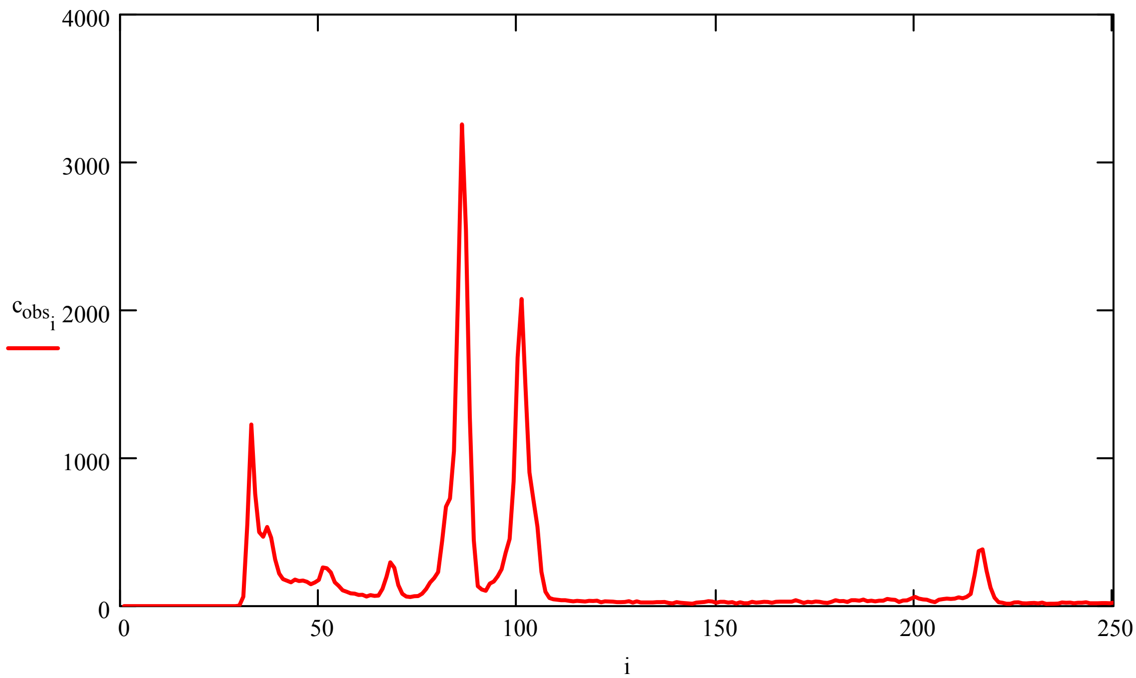
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	3	63	555
1228	760	500	469	535	462	315	220
182	171	161	179	169	172	164	148
160	177	261	256	228	161	137	107
97	86	84	75	76	64	74	69
71	116	197	296	259	142	83	64

Use the functions Col and Row to convert this data format to a more conventional two-column format.

$$i := 1 .. 250$$

$$c_{obs_i} := \beta_{\mathrm{Row}(i), \mathrm{Col}(i)}$$

This is what the 250-channel spectrum looks like.



Functions to calculate the background continuum and the peak for the 43 keV gamma rays. This assumes that the background continuum is either linear or a step function and the peak is Gaussi

$$\mathrm{bkg}(ch, bkg_{low}, bkg_{high}) := \left| \begin{array}{l} a \leftarrow bkg_{low} \;\; \text{if } ch \le c_{low} \\ a \leftarrow bkg_{high} \;\; \text{if } ch \ge c_{high} \\ a \leftarrow bkg_{low} - \dfrac{(c_{low} - ch)\cdot(bkg_{low} - bkg_{high})}{(c_{low} - c_{high})} \;\; \text{otherwise} \end{array} \right.$$

$$\mathrm{bkg}(ch, bkg_{low}, bkg_{high}) := \left| \begin{array}{l} a \leftarrow bkg_{low} \;\; \text{if } ch \le 216 \\ a \leftarrow bkg_{high} \;\; \text{otherwise} \end{array} \right.$$

$$C(ch, A, \mu, \sigma, bkg_{low}, bkg_{high}) := A\cdot \mathrm{dnorm}(ch, \mu, \sigma) + \mathrm{bkg}(ch, bkg_{low}, bkg_{high})$$

Initial guesses for the peak area (A), centroid channel (μ), and standard deviation (σ) as well as the background area in the five peaks (ending at c_{low}) below the peak (bkg_{low}) and five peaks (beginning at c_{high}) above the peak (bkg_{high}).

$A := 9\cdot 10^2 \qquad c_{low} \equiv 213$

$\mu := 216.5 \qquad c_{high} \equiv 221$

$\sigma := 1$

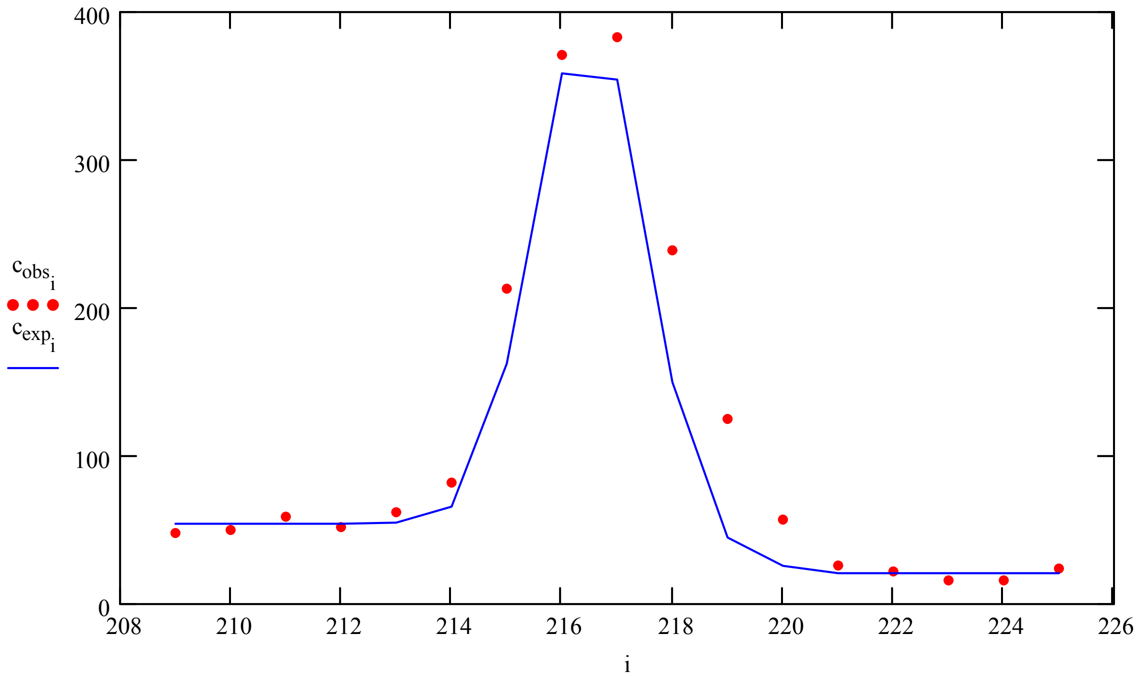
$$bkg_{low} := \frac{\left(\displaystyle\sum_{i=209}^{c_{low}} c_{obs_i}\right)}{5} \qquad bkg_{high} := \frac{\displaystyle\sum_{i=c_{high}}^{225} c_{obs_i}}{5}$$

Note that the FWHM of a peak is equal to 2.33 times the standard deviation of the peak.

View spectrum that results from initial guesses.

$i := 209..225$

$$c_{exp_i} := C(i, A, \mu, \sigma, bkg_{low}, bkg_{high})$$



Perform a non-linear weighted least squares fit of the model to the data.

Given

$$0 = \sum_{i=209}^{225} \frac{\left(c_{obs_i} - C(i, A, \mu, \sigma, bkg_{low}, bkg_{high})\right)^2}{c_{obs_i}}$$

$$\begin{pmatrix} A \\ \mu \\ \sigma \\ bkg_{low} \\ bkg_{high} \end{pmatrix} := \mathrm{Minerr}(A, \mu, \sigma, bkg_{low}, bkg_{high})$$

Weighted least squares fit estimates of parameters.

$A = 1.228 \times 10^3$ $\mu = 216.721$ $\sigma = 1.378$ $bkg_{low} = 50.745$ $bkg_{high} = 20.139$

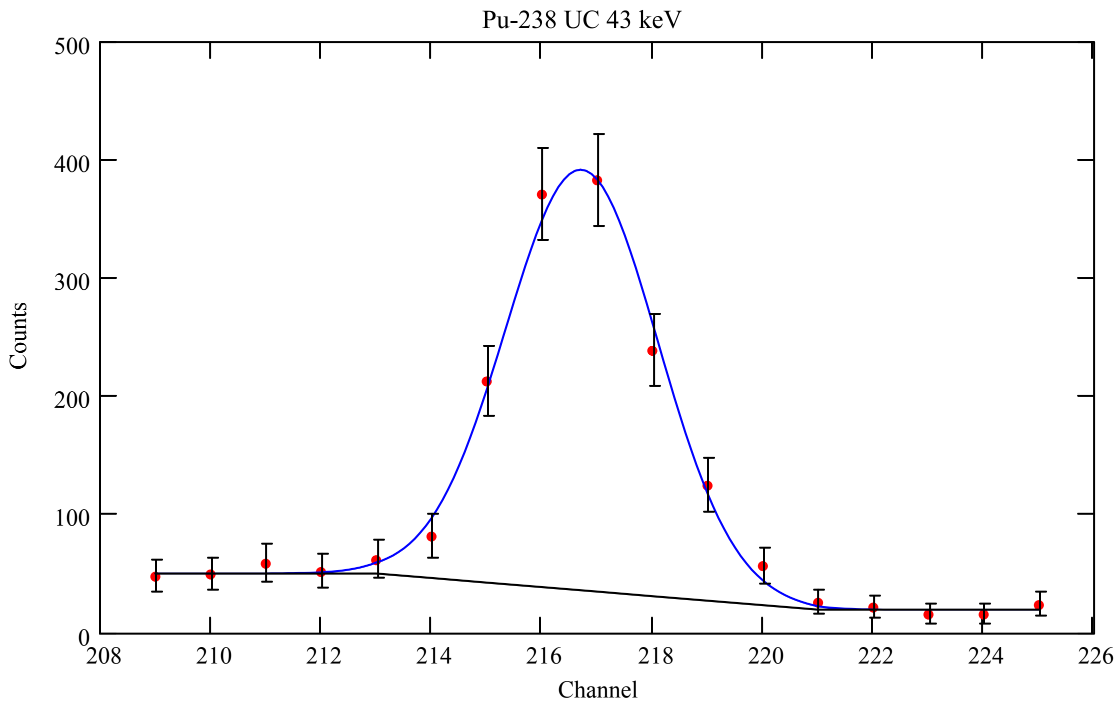
Generate the spectrum.

$j := 2090 .. 2250$

$$c_{exp_j} := C\left(\frac{j}{10}, A, \mu, \sigma, bkg_{low}, bkg_{high}\right)$$

$$c_{bkg_i} := bkg\left(i, bkg_{low}, bkg_{high}\right)$$

$$\xi := \left| \begin{array}{l} \text{for } i \in 1 .. rows(c_{obs}) \\ \quad \left| \begin{array}{l} \xi_{1,i} \leftarrow c_{obs_i} + 2\sqrt{c_{obs_i}} \\ \xi_{2,i} \leftarrow c_{obs_i} - 2\sqrt{c_{obs_i}} \end{array} \right. \\ \xi \end{array} \right.$$

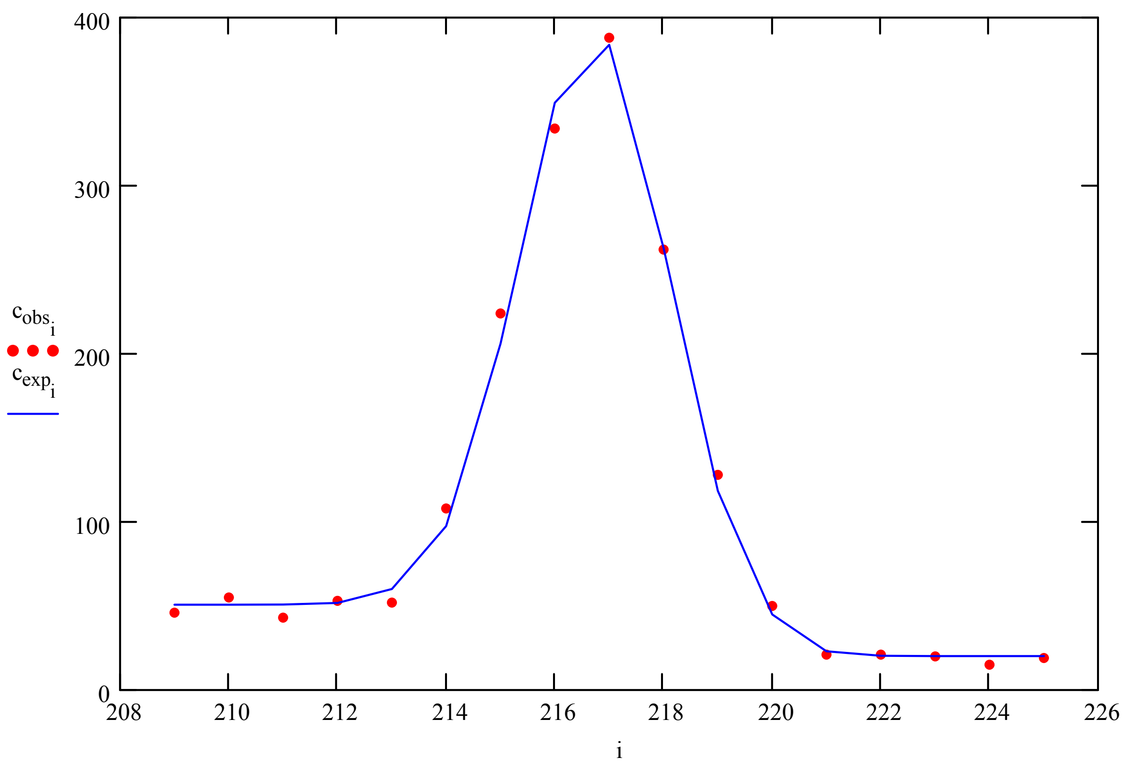


```
***************************************************************************
*****          P E A K    A N A L Y S I S    R E P O R T           *****
***************************************************************************

     Detector Name:  DET:SBDET1
     Sample Title:   UCLL/SL66P8 Lung Set (1278.2 nCi on 9/7/99)
     Peak Analysis Performed on:  4/22/2002  10:36:57 AM
               Peak Analysis From Channel:      50
               Peak Analysis To Channel:      2045

   Peak  ROI  ROI    Peak    Energy   FWHM  Net Peak  Net Area  Continuum
    No. start end  centroid   (keV)  (keV)   Area     Uncert.    Counts

F   1    48-   57    52.03    10.22   0.37  3.59E+002   56.07   1.18E+003
F   2    64-   73    68.27    13.48   0.39  5.74E+002   57.39   7.00E+002
F   3    80-   92    86.16    17.06   0.41  8.29E+003  180.09   2.19E+003
F   4    94-  107   100.98    20.03   0.43  5.63E+003  146.89   1.44E+003
F   5   208-  224   216.67    43.21   0.52  1.17E+003   69.93   5.90E+002
F   6   490-  507   497.24    99.42   0.67  2.31E+002   35.95   3.48E+002

M = First peak in a multiplet region
m = Other peak in a multiplet region
F = Fitted singlet

Errors quoted at  2.000 sigma
```

Monte Carlo estimate of uncertainty in parameters.

$$c_{exp_i} := C\left(i, A, \mu, \sigma, bkg_{low}, bkg_{high}\right)$$

$$c_{obs_i} := \text{rpois}\left(1, c_{exp_i}\right)_{1,1}$$



Create a non-linear weighted least squares fitting function.

Given

$$0 = \sum_{i=209}^{225} \frac{\left(c_{obs_i} - C\left(i, A, \mu, \sigma, bkg_{low}, bkg_{high}\right)\right)^2}{c_{exp_i}}$$

$$F\left(c_{obs}\right) := \mathrm{Minerr}\left(A, \mu, \sigma, bkg_{low}, bkg_{high}\right)$$

Repeat thirty times and average results.

$NumRuns := 30$

$$B := \left| \begin{array}{l} \text{for } j \in 1 .. NumRuns \\ \quad \left| \begin{array}{l} \text{for } i \in 209 .. 225 \\ \quad c_{obs_i} \leftarrow \mathrm{rpois}\left(1, c_{exp_i}\right)_{1,1} \\ B_j \leftarrow F\left(c_{obs}\right) \end{array} \right. \\ B \end{array} \right.$$

$$\mathrm{mean}(r) := \sum_{i=1}^{NumRuns} \frac{\left(B_i\right)_r}{NumRuns}$$

$$\mathrm{stddev}(r) := \sqrt{\sum_{i=1}^{NumRuns} \frac{\left[\left(B_i\right)_r - \mathrm{mean}(r)\right]^2}{NumRuns}}$$

$\mathrm{mean}(1) = 1.238 \times 10^3$

$\mathrm{stddev}(1) = 43.653$

$\sqrt{\mathrm{mean}(1)} = 35.186$

$\dfrac{\mathrm{stddev}(1)}{\mathrm{mean}(1)} = 0.035$

Chest Counter

Pu-238 UC Lungs - 17 keV

A 5400 second count of the University of Cincinnati lung set on the bed counting system. The lungs contain 1278.2 nCi of Pu-238 on 9/7/99. The 17 keV x-ray group will be analyzed in this worksheet.

$\mathrm{ORIGIN} \equiv 1$

Functions to convert data from ABACOS format to a conventional two-column format.

$$\mathrm{Col}(x) := \left| \begin{array}{l} \mathrm{Col} \leftarrow \mathrm{mod}(x,8) \\ \mathrm{Col} \leftarrow 8 \ \text{ if } \ \mathrm{Col} = 0 \end{array} \right.$$

$$\mathrm{Row}(x) := \left| \begin{array}{l} \mathrm{Row} \leftarrow \mathrm{trunc}\left(\frac{x}{8}\right) + 1 \\ \mathrm{Row} \leftarrow \mathrm{trunc}\left(\frac{x}{8}\right) \ \text{ if } \ \mathrm{Col}(x) = 8 \end{array} \right.$$

Counts per channel from Ge detector. Counts in $\beta_{1,1}$ are from channel 1 and counts from $\beta_{2,1}$ are from channel 9 and so on.

$\beta :=$

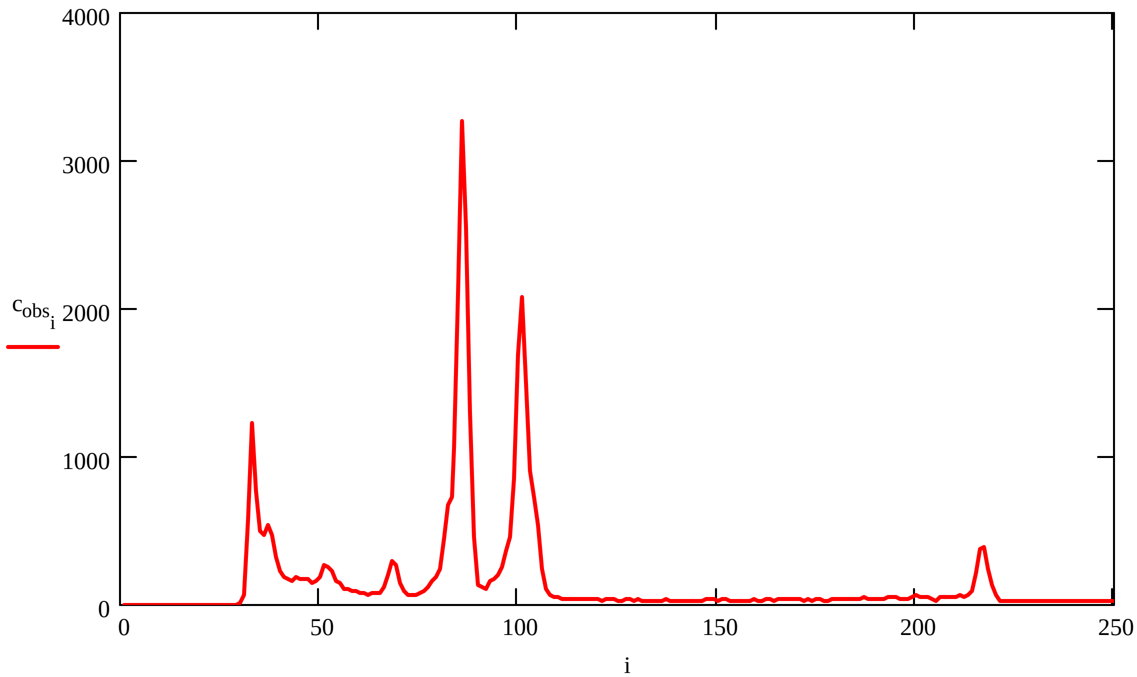
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	3	63	555
1228	760	500	469	535	462	315	220
182	171	161	179	169	172	164	148
160	177	261	256	228	161	137	107
97	86	84	75	76	64	74	69
71	116	197	296	259	142	83	64

Use the functions Col and Row to convert this format to a more conventional two-column format.

$i := 1..250$

$$c_{obs_i} := \beta_{\mathrm{Row}(i),\mathrm{Col}(i)}$$

This is what the 250-channel spectrum looks like.



Functions to calculate the background continuum and the peaks for the 17 keV group of x rays. This assumes that the background continuum is a single exponential and the peaks are Gaussian.

$$C(ch, A, \mu, \sigma, a, k) := \left| \begin{array}{l} sum \leftarrow a \cdot \exp(k \cdot ch) \\ \text{for } j \in 1..3 \\ \quad sum \leftarrow sum + A_j \cdot dnorm(ch, \mu_j, \sigma_j) \\ sum \end{array} \right.$$

Initial guesses for the peak areas (A), centroid channels (μ), and standard deviations (σ) as well as the area (a) and decay constant (k) for the single exponential continuum.

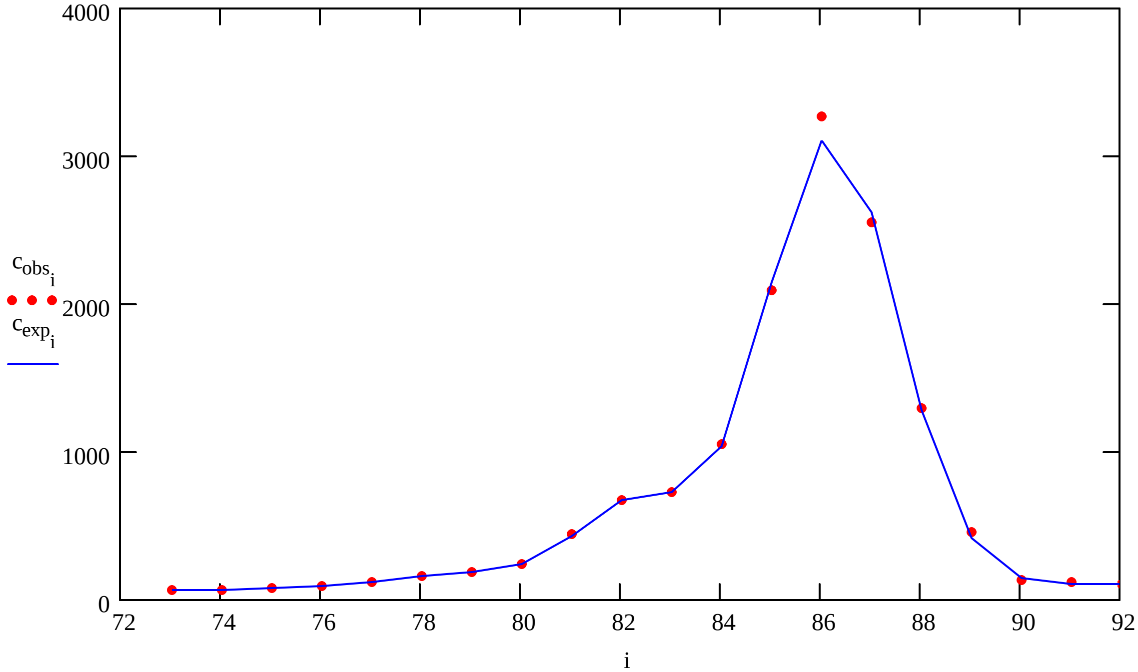
$$A := \begin{pmatrix} 3.520 \times 10^2 \\ 1.719 \times 10^3 \\ 1.008 \times 10^4 \end{pmatrix} \quad \mu := \begin{pmatrix} 78.738 \\ 82.266 \\ 86.197 \end{pmatrix} \quad \sigma := \begin{pmatrix} 1.361 \\ 1.169 \\ 1.317 \end{pmatrix} \quad a := 1.5 \cdot 10^1 \quad k := 0.02$$

Note that the FWHM of a peak is equal to 2.33 times the standard deviation of the peak.

View spectrum that results from initial guesses.

$i := 73..\ 92$

$c_{exp_i} := C(i, A, \mu, \sigma, a, k)$



Perform a non-linear weighted least squares fit of the model to the data.

Given

$$0 = \sum_{i=73}^{92} \frac{\left(c_{obs_i} - C(i, A, \mu, \sigma, a, k)\right)^2}{c_{obs_i}}$$

$$\begin{pmatrix} A \\ \mu \\ \sigma \\ a \\ k \end{pmatrix} := \mathrm{Minerr}(A, \mu, \sigma, a, k)$$

Weighted least squares fit estimates of parameters.

$$A = \begin{pmatrix} 3.276 \times 10^{2} \\ 1.722 \times 10^{3} \\ 1.011 \times 10^{4} \end{pmatrix} \qquad \mu = \begin{pmatrix} 78.722 \\ 82.249 \\ 86.197 \end{pmatrix} \qquad \sigma = \begin{pmatrix} 1.298 \\ 1.171 \\ 1.32 \end{pmatrix} \qquad a = 14.289 \qquad k = 0.021$$

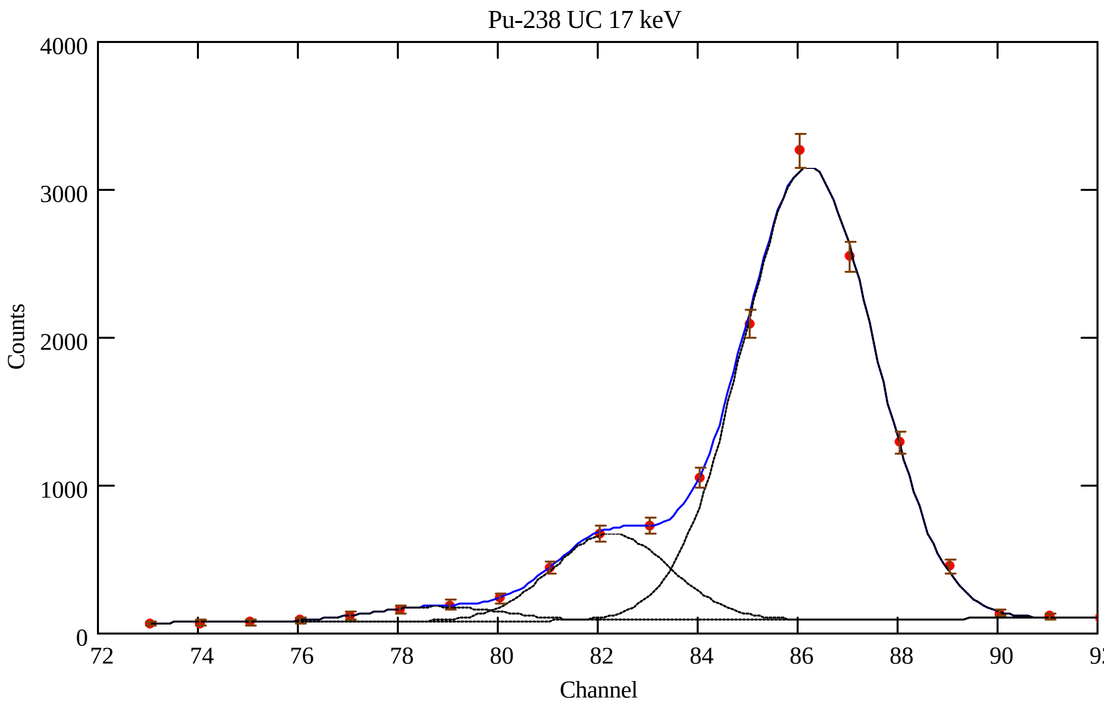
Generate the final spectrum.

$$j := 730..920$$

$$c_{exp_j} := C\left(\frac{j}{10}, A, \mu, \sigma, a, k\right) \qquad c_{17_j} := a \cdot \exp\left(k \cdot \frac{j}{10}\right) + A_3 \cdot \mathrm{dnorm}\left(\frac{j}{10}, \mu_3, \sigma_3\right)$$

$$c_{16_j} := a \cdot \exp\left(k \cdot \frac{j}{10}\right) + A_2 \cdot \mathrm{dnorm}\left(\frac{j}{10}, \mu_2, \sigma_2\right) \qquad c_{15_j} := a \cdot \exp\left(k \cdot \frac{j}{10}\right) + A_1 \cdot \mathrm{dnorm}\left(\frac{j}{10}, \mu_1, \sigma_1\right)$$

$$\xi := \left| \begin{array}{l} \text{for } i \in 1..\,\mathrm{rows}(c_{obs}) \\ \quad \left| \begin{array}{l} \xi_{1,i} \leftarrow c_{obs_i} + 2\sqrt{c_{obs_i}} \\ \xi_{2,i} \leftarrow c_{obs_i} - 2\sqrt{c_{obs_i}} \end{array} \right. \\ \xi \end{array} \right.$$



```
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*****          P E A K    A N A L Y S I S    R E P O R T           *****
*************************************************************************

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     Sample Title:   UCLL/SL66P8 Lung Set (1278.2 nCi on 9/7/99)
     Peak Analysis Performed on:  4/22/2002  10:36:57 AM
               Peak Analysis From Channel:     50
               Peak Analysis To Channel:     2045

   Peak  ROI  ROI    Peak    Energy   FWHM  Net Peak  Net Area  Continuum
    No. start end  centroid   (keV)  (keV)   Area     Uncert.    Counts

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F   2    64-   73    68.27    13.48   0.39  5.74E+002   57.39   7.00E+002
F   3    80-   92    86.16    17.06   0.41  8.29E+003  180.09   2.19E+003
F   4    94-  107   100.98    20.03   0.43  5.63E+003  146.89   1.44E+003
F   5   208-  224   216.67    43.21   0.52  1.17E+003   69.93   5.90E+002
F   6   490-  507   497.24    99.42   0.67  2.31E+002   35.95   3.48E+002

M = First peak in a multiplet region
m = Other peak in a multiplet region
F = Fitted singlet

Errors quoted at  2.000 sigma
```
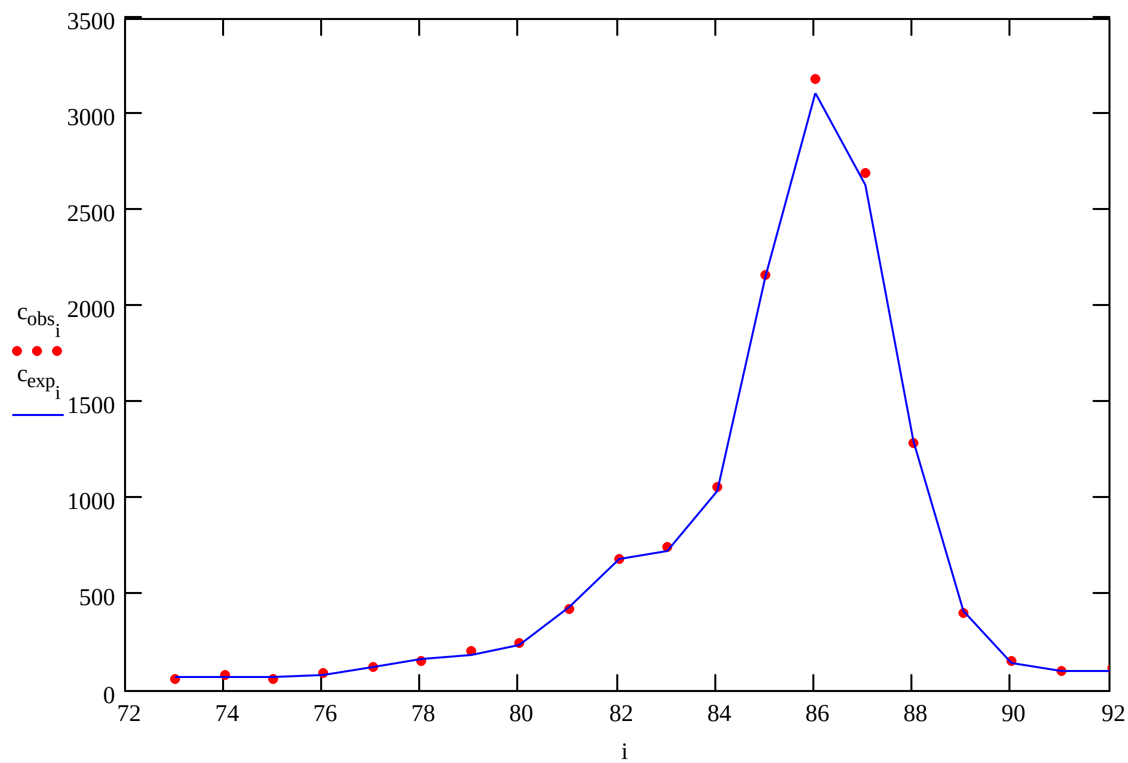

Monte Carlo estimate of uncertainty in parameters.

$$c_{exp_i} := C(i, A, \mu, \sigma, a, k)$$

$$c_{obs_i} := \mathrm{rpois}\left(1, c_{exp_i}\right)_{1,1}$$



Create a non-linear weighted least squares fitting function.

Given

$$0 = \sum_{i=73}^{92} \frac{\left(c_{obs_i} - C(i, A, \mu, \sigma, a, k)\right)^2}{c_{exp_i}}$$

$$F(c_{obs}) := \mathrm{Minerr}(A, \mu, \sigma, a, k)$$

Repeat thirty times and average results.

$$\mathrm{NumRuns} := 30$$

$$B := \left| \begin{array}{l} \text{for } j \in 1..\,\mathrm{NumRuns} \\ \quad \left| \begin{array}{l} \text{for } i \in 73..\,92 \\ \quad c_{obs_i} \leftarrow \mathrm{rpois}\left(1, c_{exp_i}\right)_{1,1} \\ B_j \leftarrow F(c_{obs}) \end{array} \right. \\ B \end{array} \right.$$

$$\mathrm{mean}(r) := \sum_{i=1}^{\mathrm{NumRuns}} \frac{\left[\left(B_i\right)_1\right]_r}{\mathrm{NumRuns}}$$

$$\mathrm{stddev}(r) := \sqrt{\sum_{i=1}^{\mathrm{NumRuns}} \frac{\left[\left[\left(B_i\right)_1\right]_r - \mathrm{mean}(r)\right]^2}{\mathrm{NumRuns}}}$$

$$\mathrm{mean}(3) = 1.014 \times 10^4$$

$$\mathrm{stddev}(3) = 140.846$$

$$\frac{\mathrm{stddev}(3)}{\mathrm{mean}(3)} = 0.014$$

Chest Counter

Pu-238 Content of DOELAP Lungs

The estimated Pu-238 activity of the DOELAP lung set relative to the UC and DOE Library lung sets and the propagated random uncertainty of the results are calculated in this worksheet using the 43 and 17 keV lines.

$$nCi \equiv 37 \cdot Bq$$

$$Activity_{UC} := 1278.2 \cdot \exp\left(\frac{-\ln(2)}{87.7 \cdot 365} \cdot 958\right) \cdot nCi \qquad Activity_{UC} = 1.252 \times 10^{3}\, nCi$$

$$Activity_{DOE} := 373 \cdot \exp\left(\frac{-\ln(2)}{87.7 \cdot 365} \cdot 5754\right) \cdot nCi \qquad Activity_{DOE} = 3.293 \times 10^{2}\, nCi$$

43 keV

Calculation of activity in DOELAP lung set relative to the University of Cincinnati lung set.

$$A := Activity_{UC} \cdot \frac{4.533 \cdot 10^{2}}{1.238 \cdot 10^{3}} \qquad A = 4.584 \times 10^{2}\, nCi$$

$$\sigma_R := \sqrt{\left(\frac{43.65}{1238}\right)^2 + \left(\frac{34.41}{453.3}\right)^2} \qquad \sigma_R = 8.370 \times 10^{-2} \qquad \sigma_R \cdot A = 3.837 \times 10^{1}\, nCi$$

$$A - \sigma_R \cdot A = 4.200 \times 10^{2}\, nCi$$

$$A + \sigma_R \cdot A = 4.968 \times 10^{2}\, nCi$$

Calculation of activity in DOELAP lung set relative to the DOE Library lung set.

$$A := Activity_{DOE} \cdot \frac{4.533 \cdot 10^{2}}{3.321 \cdot 10^{2}} \qquad A = 4.495 \times 10^{2}\, nCi$$

$$\sigma_R := \sqrt{\left(\frac{26.94}{332.1}\right)^2 + \left(\frac{34.41}{453.3}\right)^2} \qquad \sigma_R = 1.111 \times 10^{-1} \qquad \sigma_R \cdot A = 4.994 \times 10^{1}\, nCi$$

$$A - \sigma_R \cdot A = 3.995 \times 10^{2}\, nCi$$

$$A + \sigma_R \cdot A = 4.994 \times 10^{2}\, nCi$$

17 keV

Calculation of activity in DOELAP lung set relative to the University of Cincinnati lung set.

$$A := \text{Activity}_{UC} \cdot \frac{3669}{1.014 \cdot 10^{4}} \qquad A = 4.530 \times 10^{2}\,\text{nCi}$$

$$\sigma_R := \sqrt{\left(\frac{140.8}{1.014 \cdot 10^{4}}\right)^{2} + \left(\frac{124.1}{3669}\right)^{2}} \qquad \sigma_R = 3.656 \times 10^{-2} \qquad \sigma_R \cdot A = 1.656 \times 10^{1}\,\text{nCi}$$

$$A - \sigma_R \cdot A = 4.364 \times 10^{2}\,\text{nCi}$$

$$A + \sigma_R \cdot A = 4.696 \times 10^{2}\,\text{nCi}$$

Calculation of activity in DOELAP lung set relative to the DOE Library lung set.

$$A := \text{Activity}_{DOE} \cdot \frac{3669}{2356} \qquad A = 5.128 \times 10^{2}\,\text{nCi}$$

$$\sigma_R := \sqrt{\left(\frac{74.85}{2356}\right)^{2} + \left(\frac{124.1}{3669}\right)^{2}} \qquad \sigma_R = 4.640 \times 10^{-2} \qquad \sigma_R \cdot A = 2.380 \times 10^{1}\,\text{nCi}$$

$$A - \sigma_R \cdot A = 4.890 \times 10^{2}\,\text{nCi}$$

$$A + \sigma_R \cdot A = 5.366 \times 10^{2}\,\text{nCi}$$

Summary

Errors are reported at 1σ in the table and 2σ in the plot.

	DOE	UC	ABACOS
17 keV	513 ± 24	453 ± 17	474 ± 10
43 keV	449 ± 50	458 ± 38	431 ± 23