Note

MINIMUM DETECTABLE ACTIVITY WHEN BACKGROUND IS COUNTED LONGER THAN THE SAMPLE

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***Abstract*—This note discusses the use of blank or background counting data that are measured for times that differ from times used for the sample counts. The correct formula for the minimum detectable activity, under this condition, is given as follows: MDA = $[3 + 3.29\sqrt{R_b t_g(1 + t_g/t_b)}]/\epsilon t_g$, where R_b denotes background count rate, t_b and t_g denote background and gross count times, and ϵ denotes counting efficiency. Counting backgrounds for a long time reduces decision levels, uncertainties, and minimum detectable activities. These benefits are fully available only when there is no other source of variability than random fluctuations in count rates.**
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INTRODUCTION

THIS note discusses the use of blank or background counting data that are measured for times that differ from times used for the sample counts. The correct formula for the minimum detectable activity (MDA), under this condition, is given. The decision level (L_C), the sample standard deviation of the net count rate (s_n), and the MDA are all reduced through the use of long counting times, provided the only source of variability is random fluctuations in count rates. Replicate measurement of blanks provides a means of identifying nonrandom fluctuations in background count rates (Currie 1968, 1984).

THEORY

Numbers of background counts (N_b) and background counting times (t_b) are used in computing statistical values such as L_C and MDA (Currie 1968; NCRP 1985; Brodsky 1986; ANSI 1989). These two quantities are based on estimating the contribution of the background variability to the variability of the net count rate.

When the gross (or sample) counting time (t_g) is different from t_b, it is difficult to use the direct statistical formulations based on the numbers of observed counts (such as those found in many references: Currie 1968; NCRP 1985; Brodsky 1986; ANSI 1989). It is easiest to use count rates in these formulas. The background count rate is denoted by R_b ($=N_b/t_b$) and the gross count rate by R_g ($=N_g/t_g$, where N_g denotes the number of gross counts). The sample standard deviation (s_n) of the net count rate (R_n) is:

$$s_n = \sqrt{\frac{R_g}{t_g} + \frac{R_b}{t_b}}. \tag{1}$$

Currie (1968) has defined the *a posteriori* decision level for a given probability of a Type I error, α, as:

$$L_C = k_\alpha \sigma_0, \tag{2}$$

where k_α is the value of the standard normal deviate corresponding to a one-tailed probability of $1 - \alpha$, and the population standard deviation $\sigma_0 = \sigma_n$ when $R_n = 0$. The usual choice of α is 0.05, so k_α becomes 1.645. When the net counting result is observed to be equal to L_C, we say that we have detected a positive amount of activity equal to L_C with a false-positive probability of 0.05.

It is possible to estimate σ_0 by s_n evaluated at $R_n = 0$ (i.e., $R_g = R_b$):

$$\sigma_0 \approx s_n = \sqrt{\frac{R_b}{t_g} + \frac{R_b}{t_b}}. \tag{3}$$

Substituting eqn (3) into eqn (2) and rearranging, the following equation is obtained:

$$L_C = \frac{k_\alpha}{t_g}\sqrt{R_b t_g(1 + t_g/t_b)}. \tag{4}$$

Eqn (4) is valid for any combination of counting times. Note that $R_b t_g$ is the expected number of background counts during the sample counting time.

Currie (1968) defines the *a priori* detection level (L_D) as a function of k_α and k_β, where the latter is the probability of not detecting activity when it is present. Currie [1968, eqn (13)] considers the special case $k_\alpha = k_\beta = k$, in which L_D becomes:

$$L_D(R_n) = \frac{k^2}{t_g} + 2L_C(R_n). \tag{5}$$

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In eqn (5), the detection level is expressed in terms of count rates. The MDA for a net count rate is related to L_D by the reciprocal of the counting efficiency, ϵ (cps Bq^{-1}). Using Brodsky's (1986) substitution for k^2, that is, 3 instead of 2.71, the MDA becomes:

$$MDA = \frac{3 + 3.29\sqrt{R_b t_g(1 + t_g/t_b)}}{\epsilon t_g}. \quad (6)$$

Note that eqn (5) and eqn (6) do *not* assume that the standard deviation at background is the same as the standard deviation at MDA (Currie 1968), contrary to recent claims in the literature (Brodsky and Gallagher 1991).

The expression $(1 + t_g/t_b)^{1/2}$ is analogous to Currie's $(1 + 1/b)$, where Currie's b is the number of replicates of a measurement [Currie 1984, eqn (24)]. The expression $(1 + t_g/t_b)^{1/2}$ varies from $\sqrt{2}$ when $t_b = t_g$ (the "paired blank" situation, giving the familiar $4.65 = 3.29 \times \sqrt{2}$) to 1 when $t_b \gg t_g$. Several values are shown in Table 1. It is evident that merely doubling background time results in significant reduction in L_C and MDA. A greater than 10-fold increase in background count time gives diminishing returns.

Table 1. Values of $(1 + t_g/t_b)^{1/2}$ and $3.29 \times (1 + t_g/t_b)^{1/2}$ for various ratios t_b/t_g and t_g/t_b.

t_b/t_g	t_g/t_b	$(1 + t_g/t_b)^{1/2}$	$3.29 \times (1 + t_g/t_b)^{1/2}$
1	1	1.414	4.653
1.5	0.667	1.291	4.247
2	0.5	1.225	4.029
3	0.333	1.155	3.799
4	0.25	1.118	3.678
5	0.2	1.095	3.604
6	0.167	1.080	3.554
7	0.143	1.069	3.517
8	0.125	1.061	3.490
9	0.111	1.054	3.468
10	0.1	1.049	3.451
16	0.0625	1.031	3.391
20	0.05	1.025	3.371
25	0.04	1.020	3.355
50	0.02	1.010	3.323
100	0.01	1.005	3.306
1,000	0.001	1.000	3.292
10,000	0.0001	1.000	3.290

It is important to note that repeated counts of the same blank and the count times can be added, provided the experimental apparatus does not change and there is no other source of variability than the random (i.e., Poisson) fluctuations. Under these conditions, the resultant larger t_b and N_b can be used in the formulas previously given.

CONCLUSIONS

This work shows how counting backgrounds for a long time reduces decision levels, uncertainties, and MDAs—all benefits that applied health physicists should easily recognize. These benefits, of course, are fully available only when there is no other source of variability than random fluctuations in count rates.

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