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Interpretation of Low-Activity Counting

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Abstract

This report deals with the problem of estimating the unknown activity of a radioactive source. The estimate is based on the information that the radioactive source plus the background of the detector have given the count number n_1 in a certain time interval, and that the background alone has given the count number n_2 in a period of the same length. Further it is assumed that the activity of the radioactive source is positive or equal to zero.

The mean value and the standard deviation of the probability distribution of the unknown activity are derived from these assumptions, and tabulations are made and approximative expressions derived for practical use.

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Contents

Introduction

The intention of this report is to discuss the theory of interpretation of counting experiments. In such experiments the activity of the radioactive source will be the unknown parameter which has to be estimated on the basis of the observations.

In the treatment given by Bateman[1] the activity was assumed to be known, and the variance (standard deviation) of the series of count numbers from a repeated experiment was calculated. However, the observed count number from a single experiment does not in fact exhibit any spread, but the spread is attached to our estimate of the activity which was the primary source of that count number.

If the problem is to estimate the count number in a subsequent experiment with the same radioactive source, the old treatment can be used, but only after the activity of the source has been estimated. The result of this problem is then found by an integration of all possible values of the still unknown activity.

Theory of the Counting of Radioactive Decays

In order to obtain a general description of the decays in a radioactive source, the decay of one atom in the source may first be considered. We assume that each atom has a constant decay probability, λ (equal for all atoms in the source). It is defined in such a way that the probability that the atom decays in a small time interval, dt, is proportional to the length of dt and independent of how long the atom has existed. The decay probability, λ, is then the proportionality factor. We here assume that at any time the atom either exists or has decayed, so that there is no definite time interval during which the atom is decaying.

From this it is possible to give the probability $D(\lambda;t)$ that an atom with the decay probability λ decays in the time t by

$$D(\lambda;t) = \int_0^t \left\{1 - D(\lambda;\tau)\right\} \lambda d\tau \,, \tag{1}$$

$1 - D(\lambda;\tau)$ being the probability that the atom does not decay in the interval $(0, \tau)$. This equation has the solution

$$D(\lambda;t) = 1 - e^{-\lambda t}, \tag{2}$$

and we see that

$$\lim_{t \to \infty} D(\lambda; t) = 1, \tag{3}$$

which means that sooner or later the atom will decay; however, there is a positive probability, $e^{-\lambda t}$, that the atom has not decayed in the course of a finite time interval, t, however large t may be.

If a source with N equal atoms - i.e. atoms with the same decay probability, λ - is given, the calculation of the probability, $G(N, \lambda, t; n)$, that n of the N atoms decay in the time t, is an elementary problem of mathematical statistics, especially if the decay of one atom is independent of the decay of the other atoms [2]. Except for a few extraordinary cases, this assumption may be considered correct, and thus $G(N, \lambda, t; n)$ is given by a binomial distribution

$$\begin{aligned} G(N, \lambda, t; n) &= \frac{N!}{n!\,(N-n)!} (D(\lambda; t))^n (1 - D(\lambda; t))^{N-n} \\ &= \frac{N!}{n!\,(N-n)!} (1 - e^{-\lambda t})^n (e^{-\lambda t})^{N-n}, \end{aligned} \tag{4}$$

$D(\lambda; t)$ being the probability that the individual atom decays in the time t.

Normally, the detectors used for detecting radioactive decays are not 100 per cent. effective. If the efficiency of such a detector (equal to the probability that it detects the decay of a single atom) is denoted $\varepsilon \leqslant 1$, the probability, p, of the single atom decaying in the time t and being detected is given by

$$p = \varepsilon \cdot D(\lambda; t), \tag{5}$$

and the probability, $G(N, p; n_o)$, of $n_o \leqslant n \leqslant N$ of the N atoms decaying and being detected in the time t is then given by

$$G(N, p; n_o) = \frac{N!}{n_o!\,(N-n_o)!} p^{n_o} (1-p)^{N-n_o}. \tag{6}$$

Now it is easy to calculate the mean value of the count numbers observed, which appears when we repeat the experiment with known N and p. But in practice this problem very seldom occurs, and in the case of determination of the "strength" of the source it is only N which is unknown, while p is given by the efficiency of the counter, the decay constant for the atom and the observation time t, and n_o is the number of counts observed.

If it were experimentally possible to set $p = 1$, N would be identical with n_o, but this would require counting for an infinite period with a detector 100 per cent. effective.

Normally $p < 1$, and therefore all values of $N \geqslant n_o$ are possible, but not equally probable, and the task must be to determine the probability, $H(n_o, p; N)$, that the source contained exactly N atoms at the time $t = 0$ when n_o decays have been observed in the time t. As p is considered a known constant, it will in the following be stated as an index instead of a variable, i. e.

$$H(n_o, p; N) = H_p(n_o; N)$$

and

$$G(N, p; n_o) = G_p(N; n_o).$$

The method of solving this kind of problem is given by, among others, M. Annis et al.[3], who introduce the a priori probability, here P(N), that the number of atoms in the source is N independently of the number of observed counts (i. e. before any observation has been made), and the probability $Q(n_o)$ that the number of counts observed is n_o independently of N.

The simultaneous probability, $S_p(N, n_o)$, that the number of atoms at the time $t = 0$ is N and that the number of observed decays in the time t is n_o is now given in two ways:

$$\begin{aligned} S_p(N, n_o) &= Q(n_o) \cdot H_p(n_o; N) \\ &= P(N) \cdot G_p(N; n_o), \end{aligned} \tag{7}$$

from which we have

$$H_p(n_o; N) = \frac{P(N) \cdot G_p(N; n_o)}{Q(n_o)}. \tag{8}$$

If the two expressions for $S_p(N, n_o)$ are summed up over all possible values of N (i. e. $N \geqslant n_o$), we obtain

$$\sum_{N=n_o}^{\infty} Q(n_o)\cdot H_p(n_o;N) = Q(n_o)$$

$$= \sum_{N=n_o}^{\infty} P(N)\cdot G_p(N;n_o), \tag{9}$$

as, in order to be a probability, $H_p(n_o;N)$ must fulfil the condition

$$\sum_{N=n_o}^{\infty} H_p(n_o;N) = 1. \tag{10}$$

If the expression found for $Q(n_o)$ is inserted in formula (8), we have

$$H_p(n_o;N) = \frac{P(N)\cdot G_p(N;n_o)}{\sum\limits_{N=n_o}^{\infty} P(N)\cdot G_p(N;n_o)}, \tag{11}$$

where $G_p(N;n_o)$ is known (eq. 6). In order to proceed it is necessary to assume an <u>a priori</u> probability $P(N)$. We know that $P(N) = 0$ for $N < 0$; but sources produced with any number of atoms might be possible, and consequently it is reasonable to assume any value of $N \geqslant 0$ equally probable.

Hence

$$H_p(n_o;N) = \frac{G_p(N;n_o)}{\sum\limits_{N=n_o}^{\infty} G_p(N;n_o)} \tag{12}$$

$$= \frac{\binom{N}{n_o} p^{n_o}(1-p)^{N-n_o}}{\sum\limits_{N=n_o}^{\infty} \binom{N}{n_o} p^{n_o}(1-p)^{N-n_o}}$$

$$= \binom{N}{n_o} p^{n_o+1}(1-p)^{N-n_o}. \tag{13}$$

The mean value, $E[N]$, of this distribution has the direct meaning that on the basis of the observed count number n_o the probability distribution found for the true number of atoms at the time $t = 0$ has $E[N]$ as its centre of gravity. The mean value is found by

$$E[N] = \sum_{N=n_o}^{\infty} N \cdot H_p(n_o; N)$$

$$= \sum_{N=n_o}^{\infty} N \binom{N}{n_o} p^{n_o+1} (1-p)^{N-n_o}$$

$$= \frac{n_o+1}{p} - 1. \tag{14}$$

As a measure of the concentration in probability about this mean value we can use the standard deviation $D[N]$ (often written $\sigma(N)$), given by.

$$D^2[N] = E[N^2] - E^2[N]$$

$$= \frac{(n_o+2)(n_o+1)}{p^2} - 3\frac{n_o+1}{p} + 1 - \left\{\frac{(n_o+1)^2}{p^2} - 2\frac{n_o+1}{p} + 1\right\}$$

$$= \frac{n_o+1}{p^2}(1-p). \tag{15}$$

We see that

$$E[N] \rightarrow n_o$$

and $\quad D[N] \rightarrow 0$

for $\quad p \rightarrow 1.$

The Concept of Activity

If the half-life of the radioactive isotope (equal to $\ln 2/\lambda$) is very long compared with the observation time, t, then the detection probability

$$p = \varepsilon \cdot (1-e^{-\lambda t}) \tag{5}$$

will be very small, and the number of atoms in the radioactive sample has to be very large to make observation of a decay in the time t possible. In order not to operate with such very large and very small quantities, it is convenient to define the activity α of the source by

$$\begin{aligned} \alpha &= N \cdot p \\ &= N \cdot \varepsilon \cdot (1-e^{-\lambda t}) \qquad (16) \\ &\sim N \cdot \varepsilon \cdot \lambda \cdot t \end{aligned}$$

for very small values of the product $\lambda \cdot t$.

α is still a discrete variable, and the probability, $H(n;\alpha)$, of having a certain α when n decays have been observed in the time t is given by eq. (13), a synonymous relation between N and α being given by eq. (16). Thus we find

$$H(n; \alpha = N \cdot p) = \binom{N}{n} \cdot \frac{\alpha}{N}^{n+1} \cdot \left\{1 - \frac{\alpha}{N}\right\}^{N-n}. \qquad (17)$$

In the limit for large N, $H(n;\alpha)$ can be approximated by the use of the approximations

$$\binom{N}{n} \sim \frac{N^n}{n!}$$

$$\left\{1 - \frac{\alpha}{N}\right\}^{N-n} \sim e^{-\alpha},$$

and we obtain

$$\begin{aligned} H(n,\alpha) &\sim \frac{N^n}{n!} \cdot \frac{\alpha^{n+1}}{N^{n+1}} \cdot e^{-\alpha} \\ &= \frac{\alpha^n}{n!} e^{-\alpha} \cdot \frac{\alpha}{N}. \qquad (18) \end{aligned}$$

α is still a discrete variable, but for its larger values it can be replaced by a continuous variable, a, in the following way:

The probability, $H(n;a)da$, of having an activity, a, in the interval between a and a+da is defined by

$$\begin{aligned} H(n;a)da &= \sum_{\alpha > a}^{\alpha < a+da} \frac{\alpha^n}{n!} e^{-\alpha} \frac{\alpha}{N} \\ &\simeq \frac{a^n}{n!} e^{-a} \cdot \frac{a}{N} \sum_{\alpha > a}^{\alpha < a+da} 1 \qquad (19) \end{aligned}$$

for small da. The number, dN, of values of α between a and a+da is given by (16) to be

$$\sum_{\alpha > a}^{\alpha < a+da} 1 \sim \frac{N}{a}\, da;$$

hence

$$H(n;a)da = \frac{a^n}{n!}\, e^{-a}\, da. \tag{20}$$

The expression for H(n; a) looks like the expression for a Poisson distribution indicating the probability, G(a; n), of observing n decays when the activity of the source is a. It should, however, be noted that in the Poisson distribution n is the variable and a is a known constant, while in equation (20) n is the known observable and a the unknown activity in the probability function, H(n; a).

The mean value E [a] of H(n; a) is given by

$$\begin{aligned} E[a] &= \int_0^\infty a \cdot \frac{a^n}{n!}\, e^{-a}\, da \\ &= \frac{(n+1)!}{n!} \\ &= n+1. \end{aligned} \tag{21}$$

By way of comparison it should be mentioned that the mean value, E [n], of the Poisson distribution

$$G(a;n) = \frac{a^n}{n!}\, e^{-a}$$

is

$$\begin{aligned} E[n] &= \sum_{n=0}^{\infty} n \cdot \frac{a^n}{n!}\, e^{-a} \\ &= a. \end{aligned} \tag{22}$$

The difference in the problem manifests itself most explicitly in the cases of a = 0 and n = 0 expressed in the following way:

If we know that the activity $a = 0$, it follows from the Poisson distribution that the count number is also 0, as

$$g(0, 0) = 1$$

and

$$g(0, n) = 0 \quad (n \geqslant 1).$$

On the other hand, if we know that no decay has been observed in a time interval t, this does not necessarily mean that the activity is 0. The mean value of the probability function, $H(n; a)$, shows that the mathematical expectation value of the activity a is 1.

It is reasonable to use this mean value (21) as a measure of the unknown activity, a, when n decays have been observed in a given time t. The number of decays per time unit is generally used as a measure of activity (with the unit of 1 curie = $3.7 \cdot 10^{10}$ decays per second), and if n decays have been observed at the time t with a detector of detecting efficiency ε, the decay frequency, A_o, is given by

$$A_o = \frac{n+1}{\varepsilon \cdot t}. \tag{23}$$

As $\varepsilon \leqslant 1$, $1/t$ is the smallest observable value of A_o.

As a measure of the concentration in probability about the mean value, $E[a]$, we may use the standard deviation, $D[a]$, given by

$$D^2[a] = E[a^2] - E^2[a], \tag{24}$$

where

$$E[a^2] = \int_o^\infty a^2 \frac{a^n}{n!} e^{-a} da$$

$$= (n+2)(n+1).$$

By applying eq. (21) we obtain

$$D[a] = \sqrt{n+1}. \tag{25}$$

For the sake of curiosity it may be mentioned that the maximum value of the distribution function, $H(n; a)$, is reached for

$$a = n, \tag{26}$$

since

$$0 = \frac{d}{da} \frac{a^n}{n!} e^{-a}$$

$$= \left(\frac{n}{a} - 1\right) \frac{a^n}{n!} e^{-a}.$$

Derivation of H(n; a) from the Poisson Distribution

In the above analysis, $H(n; a)$ (eq. (20)) appeared as a limit function of the probability function $H_p(n_o; N)$ (eq. (13)) for large values of N and a constant product pN.

In the same limit the probability function, $G_p(N; n_o)$, of the number n_o of decays observed passes into a Poisson distribution:

$$G(a; n) = \frac{a^n}{n!} e^{-a}. \tag{27}$$

Since the Poisson distribution can be derived more generally than as a limit function of a binomial distribution[4)], it may be of interest to obtain the probability distribution, $H(n; a)$, direct on the basis of a Poisson distribution.

The probability of observing n decays in a given time interval is now given by (27), where $a \geqslant 0$ is a known parameter. The problem is to find the probability, $H(n; a)da$, of the parameter a lying in the interval $(a, a + da)$ when n decays have been observed. For this purpose the probability $Q(n)$ is introduced as the probability of observing n decays independently of the parameter (the activity) a, and the a priori probability, $P(a)da$, that a lies in the interval $(a, a + da)$ independently of an observation of decays.

The simultaneous probability, $S(n, a)da$, is now given by

$$S(n, a)da = P(a)da \cdot G(a; n)$$

$$= Q(n) \cdot H(n; a)da. \tag{28}$$

By integrating these two expressions over all possible values of a we find that

$$\int_0^\infty Q(n)\cdot H(n;a)da = Q(n)$$

$$= \int_0^\infty P(a)\cdot G(a;n)da,$$

which by insertion in (28) gives

$$H(n;a) = \frac{P(a)\cdot G(a;n)}{\int_0^\infty P(a)\cdot G(a;n)da}.$$

Here again it is assumed that all values of a $\geqslant 0$ have an <u>a priori</u> equal probability by which

$$H(n;a) = \frac{\frac{a^n}{n!}\, e^{-a}}{\int_0^\infty \frac{a^n}{n!}\, e^{-a}\, da}$$

$$= \frac{a^n}{n!}\, e^{-a}.$$

This result is given by Rainwater and Wu[5], who do not state the method of derivation.

Repetition of Measurement of a Radioactive Source with Constant Activity

If r measurements are made of the same source with constant decay frequency, A_o, over equally long time intervals, t, and with constant detector efficiency, ε, we find that the probability, $G(a; n_1, \ldots, n_r)$, of obtaining a certain set of count numbers $(n_1, \ldots, n_r)$ is the product of the probabilities of the individual count numbers. The activity of the source, a, during the time t is given by

$$a = A_o\cdot\varepsilon\cdot t. \qquad (29)$$

Here the assumption is made that the individual measurements are independent of each other, which is reasonable since the decay of the individual atoms was assumed to be independent of the decay of the other atoms, and we have

$$G(a; n_1, \ldots, n_r) = \frac{a^{n_1}}{n_1!} e^{-a} \cdot \ldots \cdot \frac{a^{n_r}}{n_r!} e^{-a}$$

$$= \frac{a^{n_1 + \ldots + n_r}}{n_1! \cdot \ldots \cdot n_r!} e^{-ra} . \tag{30}$$

The probability, $H(n_1, \ldots, n_r; a)da$, of the activity, a, lying in the interval $(a, a+da)$ can now be derived, still on the assumption that all values of $a \geqslant 0$ are a priori equally possible.

We find

$$H(n_1, \ldots, n_r; a)da = \frac{\dfrac{a^{n_1+\ldots+n_r}}{n_1! \cdot \ldots \cdot n_r!} e^{-ra} da}{\displaystyle\int_o^\infty \frac{a^{n_1+\ldots+n_r}}{n_1! \cdot \ldots \cdot n_r!} e^{-ra} da}$$

$$= \frac{(r \cdot a)^{n_1+\ldots+n_r}}{(n_1+\ldots+n_r)!} e^{-ra} dra, \tag{31}$$

which is identical with the probability of having an activity $r \cdot a$ in the interval $(ra, ra+dra)$ when $n_1+\ldots+n_r$ decays have been observed in the time rt.

Equation (31) shows that if a certain time interval, T, has been given for observation of decays in a radioactive source, no principal advantage is obtained by dividing this interval into r subintervals, since the probability distribution of the activity depends only on the total number of decays observed during the total observation time. Therefore it is a moot point whether the phrase "repetition of the measurement" is correct in this case, as it is in reality identical with a prolongation of the measuring time. In practice such a division of the existing time is often used as a means of estimating from the distribution of the individual count numbers whether the efficiency of the detector has stayed constant throughout the experiment.

H(n; a) is Asymptotically Normal

We found above that the distribution function, H(n; a), has the mean value

$$E[a] = n + 1$$

and the standard deviation

$$D[a] = \sqrt{n+1} .$$

The characteristic function, $\varphi(t)$, of the standardized variable (ref. 2),

$$\xi = \frac{a-(n+1)}{\sqrt{n+1}} , \tag{32}$$

is given by

$$\varphi(t) = E\left[e^{it\frac{a-(n+1)}{\sqrt{n+1}}}\right]$$

$$= \int_0^{\infty} e^{it\frac{a-(n+1)}{\sqrt{n+1}}} \frac{a^n}{n!} e^{-a} da$$

$$= e^{it\sqrt{n+1}} \left\{1 - \frac{it}{\sqrt{n+1}}\right\}^{-(n+1)}$$

$$= \left\{e^{\frac{it}{\sqrt{n+1}}} (1 - \frac{it}{\sqrt{n+1}})\right\}^{-(n+1)} ,$$

which for each fixed t we can write as

$$\varphi(t) = \left\{1 + \frac{t^2}{2(n+1)} + \vartheta \frac{t^3}{(n+1)^{3/2}}\right\}^{-(n+1)}$$

when choosing n sufficiently large, and for $|\vartheta| \leqslant 1$.

For $n + 1 \rightarrow \infty$, $\varphi(t) \rightarrow e^{-\frac{t^2}{2}}$, so that the distribution function, $H(n; a)$, goes towards a normal distribution with the mean value $n + 1$ and the standard deviation $\sqrt{n+1}$.

The Difference between Two Activities, a_1 and a_2

An interesting case is the one in which observation of decays in two radioactive sources has given the count numbers n_1 and n_2 respectively. The two probability distributions for the activities a_1 and a_2 are given by

$$H(n_1; a_1) \quad \text{and} \quad H(n_2; a_2)$$

respectively.

If the difference, ζ, between a_1 and a_2 is taken as a new statistical variable, we must consider the probability, $H(n_1, n_2; \zeta)d\zeta$, that $\zeta = a_1 - a_2$ lies in the interval $(\zeta, \zeta + d\zeta)$.

Here a general theorem (appendix 1) dealing with the semi-invariants for a sum of two statistical variables may be used, which says that the n^{th} semi-invariant, $\varkappa_{\zeta n}$, for a variable, ζ, given by

$$\zeta = x \cdot a_1 + y \cdot a_2$$

can be written as

$$\varkappa_{\zeta n} = x^n \varkappa_{a_1 n} + y^n \varkappa_{a_2 n},$$

where $\varkappa_{a_1 n}$ and $\varkappa_{a_2 n}$ are the n^{th} semi-invariants for the distribution functions for a_1 and a_2 respectively; x and y may have any real value.

Further, remembering that

$$\varkappa_{\zeta 1} = E[\zeta]$$

and

$$\varkappa_{\zeta 2} = D^2[\zeta],$$

we find

$$E[a_1-a_2] = (+1)E[a_1] + (-1)E[a_2]$$
$$= (n_1+1) - (n_2+1)$$
$$= n_1-n_2, \tag{33}$$

and

$$D^2[a_1-a_2] = (+1)^2 D^2[a_1] + (-1)^2 D^2[a_2]$$
$$= n_1+1 + n_2+1$$
$$= n_1+n_2 + 2. \tag{34}$$

Background Counts

The background of a detector is the number of registrations which, indistinguishable from registrations of decays in the source, occur without the source being present in the detector. These incorrect registrations have many causes, depending on the construction of the detector and the decays intended for registration. All these causes except contamination of the detector by radioactive material may in practice be eliminated by appropriate construction. To get rid of short-lived radioactive contamination, the detector should be kept isolated for a sufficiently long period, and care should be taken that the placing of the source in the detector does not again cause contamination with short-lived activities. The task under consideration is, therefore, to determine the activity of a source by means of a detector always provided with some active sources, information of which can be obtained only by observing the number of registrations caused by them.

Here it is fortunate that the Poisson distribution is additive, which means that the sum of the count numbers n_1 and n_2, both of which have been found according to a Poisson distribution, is also Poisson distributed.

Thus, if two activities, α and β, are given, they will cause the count numbers n_1 and n_2 with the probabilities

$$g(\alpha; n_1) = \frac{\alpha^{n_1}}{n_1!} e^{-\alpha}$$

and

$$g(\beta;n_2) = \frac{\beta^{n_2}}{n_2!}\,e^{-\beta}$$

respectively.

If only the sum, $N = n_1 + n_2$, is observed, the probability of obtaining a certain N is given by

$$G(\alpha,\beta;N) = \sum_{n_1=0}^{N} \frac{\alpha^{n_1}}{n_1!}\,e^{-\alpha}\cdot\frac{\beta^{N-n_1}}{(N-n_1)!}\,e^{-\beta}$$

$$= \frac{(\alpha+\beta)^N}{N!}\,e^{-(\alpha+\beta)} \sum_{n_1=0}^{N} \frac{N!}{n_1!\,(N-n_1)!}\left(\frac{\alpha}{\alpha+\beta}\right)^{n_1}\left(\frac{\beta}{\alpha+\beta}\right)^{N-n_1}$$

$$= \frac{(\alpha+\beta)^N}{N!}\,e^{-(\alpha+\beta)}. \tag{35}$$

By repeated application of this formula (35) the various activities causing the background in a detector can be combined to an activity, b (background activity), which is defined as the fictive activity causing the background. Now an activity, a, is measured with a counter with the background activity b, first by observing the count number n_1 caused by the sum of the activities, a + b, secondly by observing the count number n_2 caused by the background activity when the source is not present. To simplify the case it is assumed that the two observation periods are equally long.

The difference, $n_1 - n_2$, between the two count numbers observed is generally used as a measure of the activity a, which, as shown above, is a correct measure of the difference between two Poisson-distributed activities. But in so doing we disregard the extra information, in the case of counting with background, that the difference

$$(a+b) - b = a \tag{36}$$

is positive.

In order that this extra information may be utilized, the probability, $H(n_1, n_2; a)da$, of the activity a lying in the interval $(a, a+da)$ has to be derived on the basis that the count numbers n_1 and n_2 have been observed.

The conditional probability, $G(a, b; n_1)$, of observing n_1 when a and b are given, is found to be

$$G(a, b; n_1) = \sum_{K=0}^{n_1} \frac{a^K}{K!} e^{-a} \frac{b^{n_1-K}}{(n_1-K)!} e^{-b} \tag{37}$$

$$= \frac{(a+b)^{n_1}}{n_1!} e^{-(a+b)}, \tag{38}$$

and the probability, $G(b; n_2)$, of observing n_2

$$G(b; n_2) = \frac{b^{n_2}}{n_2!} e^{-b}. \tag{39}$$

The conditional probability of having a in the interval (b, b+db) when the numbers of counts observed are n_1 and n_2 is denoted

$$H(n_1, n_2; a, b)da\ db. \tag{40}$$

The unconditional probabilities (a priori probabilities) of having a in the interval (a, a+da) and b in the interval (b, b+db) are denoted

$$P(a)da \quad \text{and} \quad P(b)db \tag{41}$$

respectively.

The unconditional probability of observing a count number n_1 by measurement of source plus background and a count number n_2 by measurement of background only is denoted

$$Q(n_1, n_2). \tag{42}$$

The simultaneous probability, $S(n_1, n_2, a, b)$, of having a in the interval (a, a+da) and b in the interval (b, b+db) while observing n_1 and n_2 can now be written in two ways:

$$S(n_1, n_2, a, b)da\ db = Q(n_1, n_2)\cdot H(n_1, n_2; a, b)da\ db$$

$$= P(a)da\cdot P(b)db\cdot G(a, b; n_1)\cdot G(b; n_2). \tag{43}$$

If these two expressions are integrated over the definition domains of a and b ($a \geqslant 0$ and $b \geqslant 0$), we obtain

$$Q(n_1, n_2) \int_0^\infty \int_0^\infty H(n_1, n_2; a, b) da\, db = Q(n_1, n_2)$$

$$= \int_0^\infty \int_0^\infty P(a)\ P(b)\ G(a, b; n_1) \cdot G(b; n_2) da\, db. \qquad (44)$$

In order to be a probability distribution, $H(n_1, n_2; a, b) da\, db$ must be normalized. Now it is possible to give an expression for this probability; we find

$$H(n_1, n_2; a, b) = \frac{P(a)\ P(b)\ G(a, b; n_1)\ G(b; n_2)}{\int_0^\infty \int_0^\infty P(a)\ P(b)\ G(a, b; n_1)\ G(b; n_2) da\, db}. \qquad (45)$$

Here again it is reasonable to assume a priori that every positive value of a and b is equally probable, whereby we obtain

$$H(n_1, n_2; a, b) = \frac{G(a, b; n_1)\ G(b; n_2)}{\int_0^\infty \int_0^\infty G(a, b; n_1)\ G(b; n_2) da\, db}$$

$$= \frac{\frac{(a+b)^{n_1}}{n_1!} e^{-(a+b)} \frac{b^{n_2}}{n_2!} e^{-b}}{\int_0^\infty \int_0^\infty \frac{(a+b)^{n_1}}{n_1!} e^{-(a+b)} \cdot \frac{b^{n_2}}{n_2!} e^{-b}\, da\, db}. \qquad (46)$$

Calculating the denominator in (46) by using expression (37) for $G(a, b; n_1)$, we have

$$\int_0^\infty \int_0^\infty \frac{(a+b)^{n_1}}{n_1!} e^{-(a+b)} \frac{b^{n_2}}{n_2!} e^{-b}\, da\, db$$

$$= \sum_{K=0}^{n_1} \int_0^\infty \frac{a^K}{K!} e^{-a}\, da \int_0^\infty \frac{b^{n_1 - K + n_2}}{(n_1 - K)!\, n_2!} e^{-2b}\, db$$

$$= \sum_{K=0}^{n_1} \frac{(n_1 - K + n_2)!}{(n_1 - K)!\, n_2!\, 2^{n_1 - K + n_2 + 1}}. \qquad (47)$$

Since only the magnitude of a is of general interest, we integrate $H(n_1, n_2; a, b)$ over all values of $b \geqslant 0$, thus obtaining the conditional probability distribution, $H(n_1, n_2; a)$, for a based on the observation of the count numbers n_1 and n_2:

$$H(n_1, n_2; a) = \int_0^\infty H(n_1, n_2; a, b)db$$

$$= \frac{\sum_{K=0}^{n_1} \frac{a^K}{K!} e^{-a} \frac{(n_1 - K + n_2)!}{(n_1 - K)! \, n_2! \, 2^{n_1 - K + n_2 + 1}}}{\sum_{K=0}^{n_1} \frac{(n_1 - K + n_2)!}{(n_1 - K)! \, n_2! \, 2^{n_1 - K + n_2 + 1}}}$$

$$= \frac{\sum_{K=0}^{n_1} \frac{a^K}{K!} e^{-a} \frac{(n_1 - K + n_2)!}{(n_1 - K)!} 2^K}{\sum_{K=0}^{n_1} \frac{(n_1 - K + n_2)!}{(n_1 - K)!} 2^K} . \qquad (48)$$

Mean Value and Standard Deviation in the Case with a Background

The mean value, $E[a]$, of the probability distribution $H(n_1, n_2; a)$ is found to be

$$E[a] = \int_0^\infty a \cdot H(n_1, n_2; a)da$$

$$= \frac{\sum_{K=0}^{n_1} \int_0^\infty \frac{a^{K+1}}{K!} e^{-a} da \cdot \frac{(n_1 - K + n_2)! \, 2^K}{(n_1 - K)!}}{\sum_{K=0}^{n_1} \frac{(n_1 - K + n_2)! \, 2^K}{(n_1 - K)!}}$$

$$= \frac{\sum_{K=0}^{n_1} (K+1) \frac{(n_1 - K + n_2)! \, 2^K}{(n_1 - K)!}}{\sum_{K=0}^{n_1} \frac{(n_1 - K + n_2)! \, 2^K}{(n_1 - K)!}} . \qquad (49)$$

The expression for $E[a]$ can be calculated on a digital computer such as DASK, and in table 1 $E[a]$ is given for values of n_1 and n_2 up to 51.

It is desirable, however, to make a certain analysis of the expression and especially to investigate the limit for large values of n_1.

Simplified, the expression can be written

$$E[a] = \frac{\sum_{K=0}^{n_1} (K+1)\, f(K)}{\sum_{K=0}^{n_1} f(K)}, \tag{50}$$

where $f(K) > 0$, and as the addends in the numerator are multiplied by a factor $(K+1) \geqslant 1$, it is obvious that

$$E[a] \geqslant 1, \tag{51}$$

where the sign of equality holds only for $n_1 = 0$ independently of n_2.

To investigate $E[a]$ for large values of n_1 with constant n_2 we substitute r for $n_1 - K$ in the expression (49), and obtain

$$E[a] = n_1 - n_2 + \frac{\dfrac{(n_1+n_2+1)!}{n_1!\, 2^{n_1}}}{\sum_{r=0}^{n_1} \dfrac{(r+n_2)!}{r!\, 2^r}}, \tag{52}$$

and for each constant n_2 it can be found (appendix 2) that

$$\frac{\dfrac{(n_1+n_2+1)!}{n_1!\, 2^{n_1}}}{\sum_{r=0}^{n_1} \dfrac{(r+n_2)!}{r!\, 2^r}} \rightarrow 0 \quad \text{for } n_1 \rightarrow \infty, \tag{53}$$

from which the measure generally used for the source strength a is obtained, since then

$$E[a] \rightarrow n_1 - n_2. \tag{54}$$

The standard deviation, $D[a]$, given by

$$D^2[a] = E[a^2] - E^2[a]$$

is used as a measure of the concentration in probability about the mean value, $E[a]$, $E[a^2]$ being found by means of the expression

$$E[a^2] = \int_0^\infty a^2 \, H(n_1, n_2; a)da$$

$$= \frac{\sum_{K=0}^{n_1} \int_0^\infty \frac{a^{K+2}}{K!} e^{-a} da \frac{(n_1 - K + n_2)! \, 2^K}{(n_1 - K)!}}{\sum_{K=0}^{n_1} \frac{(n_1 - K + n_2)! \, 2^K}{(n_1 - K)!}}$$

$$= \frac{\sum_{K=0}^{n_1} (K+1)(K+2) \frac{(n_1 - K + n_2)! \, 2^K}{(n_1 - K)!}}{\sum_{K=0}^{n_1} \frac{(n_1 - K + n_2)! \, 2^K}{(n_1 - K)!}} . \qquad (55)$$

From this, $D^2[a]$ is found (appendix 2) to be

$$D^2[a] = n_1 + n_2 + 3 - (n_1 - n_2) \cdot \frac{\frac{(n_1 + n_2 + 1)!}{n_1! \, 2^{n_1}}}{\sum_{r=0}^{n_1} \frac{(r + n_2)!}{r! \, 2^r}}$$

$$- \left\{ \frac{\frac{(n_1 + n_2 + 1)!}{n_1! \, 2^{n_1}}}{\sum_{r=0}^{n_1} \frac{(r + n_2)!}{r! \, 2^r}} \right\}^2 \qquad (56)$$

$$\rightarrow n_1 + n_2 + 3 \quad \text{for } n_1 \rightarrow \infty \text{ and constant } n_2.$$

Table 1 also gives values of the relative standard deviation $D[a]/E[a]$ for n_1 and n_2 up to 51.

In the limit $n_1 \rightarrow 0$ it is seen from formula (56) that

$$\lim_{n_1 \rightarrow 0} D^2[a] = n_2 + 3 - (-n_2)\cdot(n_2+1) - (n_2+1)^2$$

$$= 1,$$

and it has already been shown (formula 50) that

$$\lim_{n_1 \rightarrow 0} E[a] = 1,$$

which means that the relative standard deviation

$$\frac{D[a]}{E[a]} \rightarrow 1 \quad \text{when} \quad n_1 \rightarrow 0 \qquad (57)$$

independently of the value of n_2.

Approximative Expressions for $E[a]$ and $D[a]/E[a]$

The mean value $E[a]$ of the probability distribution for the activity a was given by equation (52):

$$E[a] = n_1 - n_2 + \frac{\dfrac{(n_1+n_2+1)!}{n_1!\,2^{n_1}}}{\sum\limits_{r=0}^{n_1} \dfrac{(r+n_2)!}{r!\,2^r}}, \qquad (52)$$

which was the exact expression on the basis of the theory given.

In order to derive a more convenient formula we transform the denominator (appendix 3) into

$$\sum_{r=0}^{n_1} \frac{(r+n_2)!}{r!\,2^r} = n_2!\,2^{n_2+1} \cdot \int_0^{\frac{1}{2}} \frac{(n_1+n_2+1)!}{n_1!\,n_2!}\, x^{n_2}(1-x)^{n_1}\, dx, \qquad (58)$$

where the integrand

$$\frac{(n_1+n_2+1)!}{n_1!\,n_2!}\, x^{n_2}(1-x)^{n_1} = \beta(p, q; x)$$

is the Beta-distribution (ref. 2) with the parameters

$$p = n_2 + 1 \quad \text{and} \quad q = n_1 + 1.$$

For larger values of p and q this distribution function tends to the normal distribution (ref. 2), which can then be used in deriving an approximation, since

$$E\,[a] = n_1 - n_2 + \frac{\frac{1}{2}\,\frac{(n_1+n_2+1)!}{n_1!\,n_2!}\,(\frac{1}{2})^{n_1}\,(\frac{1}{2})^{n_2}}{\int_o^{\frac{1}{2}} \frac{(n_1+n_2+1)!}{n_1!\,n_2!}\, x^{n_2}(1-x)^{n_1}\,dx}$$

$$\sim n_1 - n_2 + \frac{\frac{1}{2\sigma}\,\frac{1}{\sqrt{2\pi}}\, e^{-\frac{x^2}{2}}}{\int_{-\infty}^{x} \frac{1}{\sqrt{2\pi}}\, e^{-\frac{t^2}{2}}\,dt}, \tag{59}$$

where

$$\sigma^2 = \frac{(n_1+1)(n_2+1)}{(n_1+n_2+2)^2(n_1+n_2+3)} \tag{60}$$

and

$$x = \frac{1}{\sigma}\,(\frac{1}{2} - \frac{n_2+1}{n_1+n_2+2}). \tag{61}$$

The approximation is only of interest when n_1 and n_2 are fairly close to each other, and then

$$\frac{1}{\sigma^2} = (1 + \frac{n_2+1}{n_1+1})(1 + \frac{n_1+1}{n_2+1})(n_1+n_2+3)$$

$$= (2 + y + \frac{1}{y})(n_1+n_2+3)$$

$$\sim 4(n_1+n_2+3), \tag{62}$$

where

$$y = \frac{n_2+1}{n_1+1}.$$

The error introduced by this approximation is seen to be of second order in y.

Now the final approximation for the mean value of the activity a is found to be

$$E[a] \sim n_1 - n_2 + \sqrt{n_1+n_2+3} \cdot \varkappa(x), \tag{63}$$

where

$$x = \frac{n_1-n_2}{\sqrt{n_1+n_2+3}} \tag{64}$$

and

$$\varkappa(x) = \frac{\frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}}}{\int_{-\infty}^{x} \frac{1}{\sqrt{2\pi}} e^{-\frac{t^2}{2}} dt}. \tag{65}$$

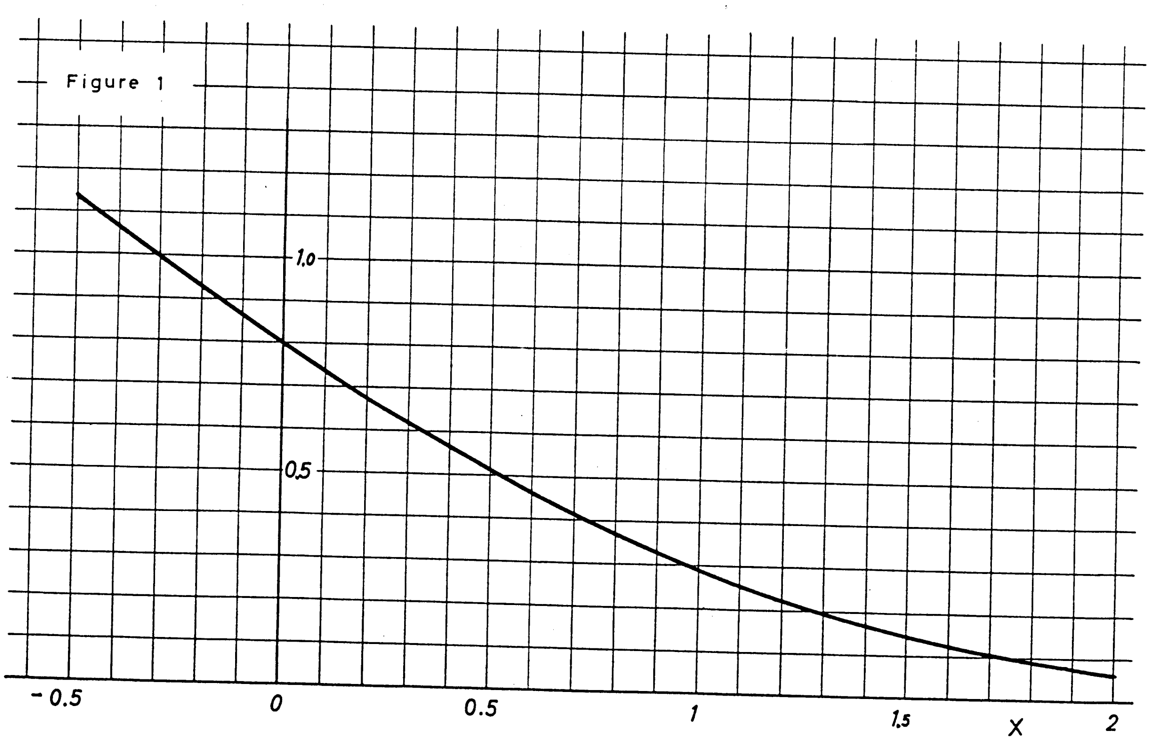

Figure 1, calculated on the basis of a standard table of the error function (e. g. ref. 6), shows the value of $\varkappa(x)$ for the range $-0.5 < x < 2.0$.

The formula for the squared standard deviation,

$$D^2[a] = n_1+n_2+3 - (n_1-n_2)\,\frac{\dfrac{(n_1+n_2+1)!}{n_1!\,2^{n_1}}}{\displaystyle\sum_{r=0}^{n_1}\frac{(r+n_2)!}{r!\,2^r}}$$

$$- \left\{ \frac{\dfrac{(n_1+n_2+1)!}{n_1!\,2^{n_1}}}{\displaystyle\sum_{r=0}^{n_1}\frac{(r+n_2)!}{r!\,2^r}} \right\}^2 , \qquad (66)$$

is found in appendix 2. The introduction of the same approximations as those used for $E[a]$ will give

$$D^2[a] \sim n_1+n_2+3 - (n_1-n_2)\sqrt{n_1+n_2+3}\,\varkappa(x) - (n_1+n_2+3)\,\varkappa^2(x), \quad (67)$$

and by division by the expression for $E[a]$ we obtain

$$\frac{D[a]}{E[a]} \sim \frac{\sqrt{1-x\,\varkappa(x)-\varkappa^2(x)}}{x+\varkappa(x)} . \qquad (68)$$

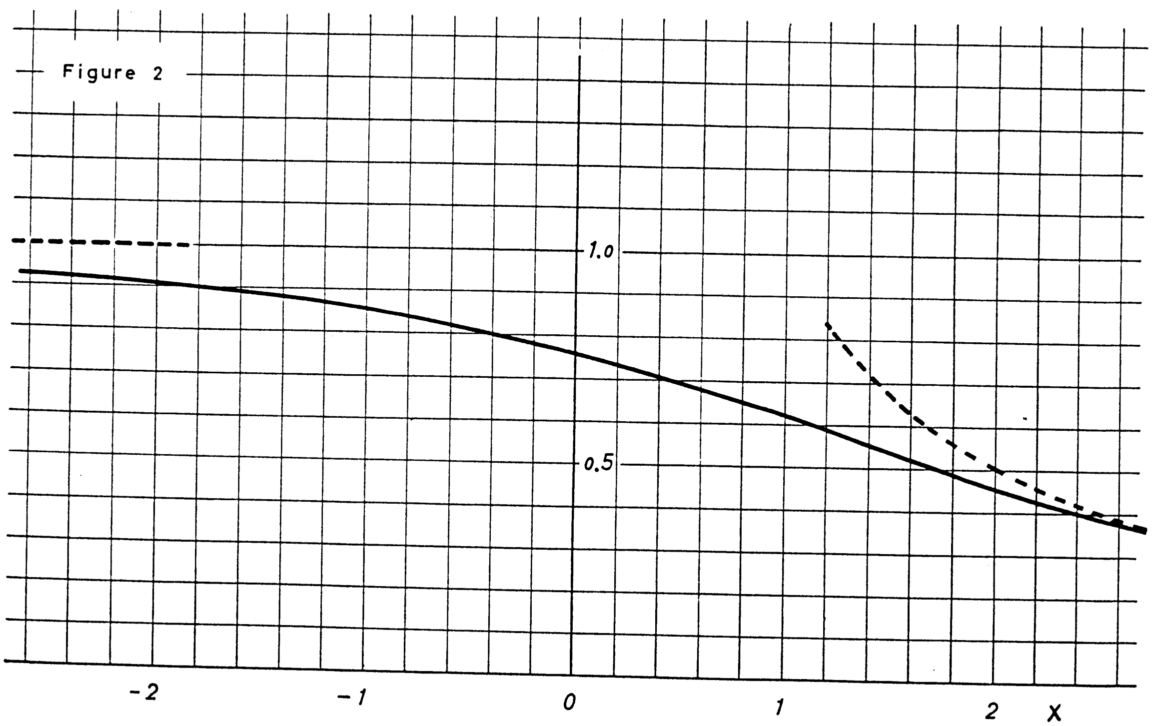

Figure 2

Figure 2, together with formula (68), shows the function x^{-1} (dashed line) since for constant n_2

$$\frac{D[a]}{E[a]} \rightarrow \frac{\sqrt{n_1+n_2+3}}{n_1-n_2} = \frac{1}{x} \quad \text{as} \quad n_1 \rightarrow \infty. \tag{69}$$

Summary

After an analysis of a set of assumptions which leads to the Poisson distribution for the observation of radioactive decays, the problem of the magnitude of the apparent activity of the radioactive source is considered.

It is assumed that the decays in the source in question are observed by means of a detector that exhibits a certain background count rate. During equal periods of observation the radioactive source plus the background are assumed to have given the number of counts n_1 and the background alone the number n_2. On this basis the probability,

$$H(n_1, n_2; a)da,$$

of the unknown activity a_1 lying in the interval $(a, a+da)$ is derived (formula 48):

$$H(n_1, n_2; a) = \frac{\sum_{K=0}^{n_1} \frac{a^K}{K!} e^{-a} \frac{(n_1-K+n_2)!}{(n_1-K)!} 2^K}{\sum_{K=0}^{n_1} \frac{(n_1-K+n_2)!}{(n_1-K)!} 2^K}.$$

It is further shown that no extra information is gained on the magnitude of the activity by a division of the observation time.

In order to find a representative measure of the activity, the mean value $E[a]$ and the standard deviation $D[a]$ of the above-mentioned probability function are derived (formulae 52 and 56 respectively):

$$E[a] = n_1 - n_2 + \frac{\frac{(n_1+n_2+1)!}{n_1!\,2^{n_1}}}{\sum_{r=0}^{n_1} \frac{(r+n_2)!}{r!\,2^r}}$$

and

$$D^2\,[a] = n_1+n_2+3 - (n_1-n_2)\cdot \frac{\dfrac{(n_1+n_2+1)!}{n_1!\,2^{n_1}}}{\sum\limits_{r=0}^{n_1} \dfrac{(r+n_2)!}{r!\,2^r}}$$

$$- \left\{ \frac{\dfrac{(n_1+n_2+1)!}{n_1!\,2^{n_1}}}{\sum\limits_{r=0}^{n_1} \dfrac{(r+n_2)!}{r!\,2^r}} \right\}^2 .$$

Since these formulae are rather inconvenient even for very small count numbers, a tabulation of $E\,[a]$ and $D\,[a]\,/E\,[a]$ is made for n_1 and n_2, both up to 51 (table 1).

Approximative expressions are derived, which give the mean value of the activity

$$E\,[a] \sim n_1-n_2 + \sqrt{n_1+n_2+3}\cdot \varkappa(x),$$

where

$$x = \frac{n_1-n_2}{\sqrt{n_1+n_2+3}}$$

and

$$\varkappa(x) = \frac{\dfrac{1}{\sqrt{2\pi}}\, e^{-\frac{x^2}{2}}}{\displaystyle\int_{-\infty}^{x} \frac{1}{\sqrt{2\pi}}\, e^{-\frac{t^2}{2}}\, dt} .$$

The relative standard deviation is found to be approximated by

$$\frac{D\,[a]}{E\,[a]} \sim \frac{\sqrt{1-x\,\varkappa(x)-\varkappa^2(x)}}{x+\varkappa(x)} .$$

$\varkappa(x)$ can be taken from fig. 1 and $D\,[a]\,/E\,[a]$ from fig. 2.

When $n_1 \gg n_2$ in such a way that x is large, it is found that

$$E[a] \rightarrow n_1 - n_2$$

and

$$\frac{D[a]}{E[a]} \rightarrow \frac{1}{x} = \frac{\sqrt{n_1 + n_2 + 3}}{n_1 - n_2}.$$

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The report has been carefully typed by Mrs. Karna Hansen, the figures have been drawn by Kurt Holmqvist, and special signs in the text have been written by Knud J. Nielsen.

The heavy burden of keeping the text in the vicinity of correct English grammar has been carried bravely by Flemming Steenbuch.

Appendix 1

Theorem on Semi-invariants

If a statistical variable, ξ , is given as a weighed sum of two independent statistical variables, η and ζ, by

$$\xi = a \cdot \eta + b\zeta ,$$

where a and b are real numbers, the n^{th} semi-invariant $\varkappa_{n,\xi}$ of ξ is

$$\varkappa_{n,\xi} = a^n \varkappa_{n,\eta} + b^n \varkappa_{n,\zeta} ,$$

where $\varkappa_{n,\eta}$ and $\varkappa_{n,\zeta}$ are the n^{th} semi-invariants of η and ζ respectively.

Proof:

The characteristic function, $\varphi_\xi(t)$, of the statistical variable ξ (ref. 2) is expressed by

$$\varphi_\xi(t) = \varphi_\eta(t) \cdot \varphi_\zeta(t)$$

since η and ζ were independent variables. A power-series expansion of the logarithm of the characteristic function yields

$$\begin{aligned} \log \varphi_\xi(t) &= \sum_{n=1}^{k} \frac{\varkappa_{n,\xi}}{n!} (it)^n + \sigma(t^k) \\ &= \log \varphi_\eta(t) + \log \varphi_\zeta(t) \\ &= \sum_{n=1}^{k} \frac{\varkappa_{n,\eta}}{n!} (iat)^n + \sigma(t^k) + \sum_{n=1}^{k} \cdot \frac{\varkappa_{n,\zeta}}{n!} (ibt)^n + \sigma(t^k). \end{aligned}$$

Hence

$$\sum_{n=1}^{k} \frac{\varkappa_{n,\xi}}{n!} (it)^n + \sigma(t^k) = \sum_{n=1}^{k} \frac{\left(a^n \varkappa_{n,\eta} + b \varkappa_{n,\zeta}\right)}{n!} (it)^n + \sigma(t^k).$$

The identity of the coefficients to the same power of the variable immediately yields the theorem.

Appendix 2

Calculation of Expressions for E [a] and D [a]

The expression for the mean value of the activity as given by formula (49),

$$E[a] = \frac{\sum_{K=0}^{n_1} (K+1) \frac{(n_1-K+n_2)!\,2^K}{(n_1-K)!}}{\sum_{K=0}^{n_1} \frac{(n_1-K+n_2)!\,2^K}{(n_1-K)!}}, \qquad (49)$$

is transformed by the substitution $r = n_1 - K$ into

$$E[a] = \frac{\sum_{r=0}^{n_1} (n_1+1-r) \frac{(r+n_2)!}{r!} 2^{n_1-r}}{\sum_{r=0}^{n_1} \frac{(r+n_2)!}{r!} 2^{n_1-r}}$$

$$= n_1+1 - \frac{\sum_{r=0}^{n_1} r \frac{(r+n_2)!}{r!\,2^r}}{\sum_{r=0}^{n_1} \frac{(r+n_2)!}{r!\,2^r}}$$

$$= n_1 - n_2 + \frac{\frac{(n_1+n_2+1)!}{n_1!\,2^{n_1}}}{\sum_{r=0}^{n_1} \frac{(r+n_2)!}{r!\,2^r}} .$$

In the last step we use the relation

$$\sum_{r=0}^{n_1} r \frac{(r+n_2)!}{r!\,2^r} = \sum_{r=1}^{n_1} \frac{(r+n_2)(r-1+n_2)!}{(r-1)!\,2^{r-1}\cdot 2}$$

$$= \frac{1}{2} \sum_{r=1}^{n_1} \frac{(r-1+n_2+1)(r-1+n_2)!}{(r-1)!\,2^{r-1}}$$

$$= \frac{1}{2}\left\{ \sum_{r=1}^{n_1} (r-1) \frac{(r-1+n_2)!}{(r-1)!\,2^{r-1}} + (n_2+1) \sum_{r=1}^{n_1} \frac{(r-1+n_2)!}{(r-1)!\,2^{r-1}} \right\}$$

$$= \frac{1}{2}\left\{ \sum_{r=0}^{n_1-1} r \frac{(r+n_2)!}{r!\,2^r} + (n_2+1) \sum_{r=0}^{n_1-1} \frac{(r+n_2)!}{r!\,2^r} \right\}$$

$$= \frac{1}{2}\left\{ \sum_{r=0}^{n_1} r \frac{(r+n_2)!}{r!\,2^r} + (n_2+1) \sum_{r=0}^{n_1} \frac{(r+n_2)!}{r!\,2^r} - n_1 \frac{(n_1+n_2)!}{n_1!\,2^{n_1}} - (n_2+1) \frac{(n_1+n_2)!}{n_1!\,2^{n_1}} \right\}$$

$$= (n_2+1) \sum_{r=0}^{n_1} \frac{(r+n_2)!}{r!\,2^r} - \frac{(n_1+n_2+1)!}{n_1!\,2^{n_1}} \; .$$

The preceding calculations are somewhat facilitated by the use of the abbreviations

$$X = \sum_{r=0}^{n_1} \frac{(r+n_2)!}{r!\,2^r}$$

and

$$Y = \frac{(n_1+n_2+1)!}{n_1!\,2^{n_1}} ,$$

by which

$$E[a] = n_1 - n_2 + \frac{Y}{X}.$$

In order to show that

$$\frac{Y}{X} = \frac{\dfrac{(n_1+n_2+1)!}{n_1!\,2^{n_1}}}{\displaystyle\sum_{r=0}^{n_1} \frac{(r+n_2)!}{r!\;2^r}} \rightarrow 0 \qquad \text{for } n_1 \rightarrow \infty \text{ and } n_2 \text{ constant}$$

we first show that the denominator is non-vanishing. Since the denominator is a sum of positive elements, it is bigger than e. g. the first of the addends:

$$\sum_{r=0}^{n_1} \frac{(r+n_2)!}{r!\,2^r} \gg \frac{(0+n_2)!}{0!\,2^0} = n_2!\ ,$$

which is non-vanishing when $n_2 \gg 1$.

Using Sterling's formula

$$n! \sim e^{-n} \cdot n^n \cdot \sqrt{2\pi n}\,,$$

we then see that the numerator tends to zero for $n_1 \rightarrow \infty$ and n_2 constant, since

$$\frac{(n_1+n_2+1)!}{n_1!\,2^{n_1}} \sim \frac{e^{-(n_1+n_2+1)}\,(n_1+n_2+1)^{(n_1+n_2+1)}\,\sqrt{2\pi\,(n_1+n_2+1)}}{e^{-n_1}\,n_1^{\,n_1}\,\sqrt{2\pi n_1}\;\,2^{n_1}}$$

$$= \frac{1}{e^{n_2+1}} \left(\left\{ 1 + \frac{n_2+1}{n_1} \right\}^{\frac{n_1}{n_2+1}+1} \right)^{n_2+1} \frac{n_1^{\,n_2+1}}{2^{n_1}} \sqrt{1 + \frac{n_2+1}{n_1}}$$

$$\frac{(n_1+n_2+1)!}{n_1!\,2^{n_1}} \rightarrow \frac{1}{e^{n_2+1}} \cdot e^{n_2+1} \cdot 0 \cdot 1$$

$$= 0$$

for $n_1 \rightarrow \infty$ and n_2 constant, which concludes the proof.

By substituting r for n_1 - K we transform the expression

$$E\,[a^2] = \frac{\sum\limits_{K=0}^{n_1} (K+1)(K+2)\,\frac{(n_1-K+n_2)!\,2^K}{(n_1-K)!}}{\sum\limits_{K=0}^{n_1} \frac{(n_1-K+n_2)!\,2^K}{(n_1-K)!}}$$

into

$$E\,[a^2] = \frac{\sum\limits_{r=0}^{n_1} (n_1+1-r)(n_1+2-r)\,\frac{(r+n_2)!\,2^{n_1-r}}{r!}}{\sum\limits_{r=0}^{n_1} \frac{(r+n_2)!\,2^{n_1-r}}{r!}}$$

$$= (n_1+1)(n_1+2) - (2n_1+3)\,\frac{\sum\limits_{r=0}^{n_1} r\,\frac{(r+n_2)!}{r!\,2^r}}{\sum\limits_{r=0}^{n_1} \frac{(r+n_2)!}{r!\,2^r}}$$

$$+ \frac{\sum\limits_{r=0}^{n_1} r^2\,\frac{(r+n_2)!}{r!\,2^r}}{\sum\limits_{r=0}^{n_1} \frac{(r+n_2)!}{r!\,2^r}} .$$

Here we find that

$$\sum_{r=0}^{n_1} r^2\,\frac{(r+n_2)!}{r!\,2^r} = \sum_{r=1}^{n_1} r\,\frac{(r+n_2)!}{(r-1)!\,2^r}$$

$$= \sum_{r=1}^{n_1} (r-1+1)(r-1+n_2+1)\,\frac{(r-1+n_2)!}{(r-1)!\,2^{r-1}\cdot 2}$$

$$= \frac{1}{2}\left\{\sum_{r=1}^{n_1}(r-1)^2\frac{(r-1+n_2)!}{(r-1)!\,2^{r-1}} + (n_2+2)\sum_{r=1}^{n_1}(r-1)\frac{(r-1+n_2)!}{(r-1)!\,2^{r-1}}\right.$$

$$\left. + (n_2+1)\sum_{r=1}^{n_1}\frac{(r-1+n_2)!}{(r-1)!\,2^{r-1}}\right\}$$

$$= \frac{1}{2}\left\{\sum_{r=0}^{n_1} r^2\frac{(r+n_2)!}{r!\,2^r} + (n_2+2)\sum_{r=0}^{n_1} r\frac{(r+n_2)!}{r!\,2^r} + (n_2+1)\sum_{r=0}^{n_1}\frac{(r+n_2)!}{r!\,2^r}\right.$$

$$\left. - n_1^2\frac{(n_1+n_2)!}{n_1!\,2^{n_1}} - (n_2+2)\cdot n_1\frac{(n_1+n_2)!}{n_1!\,2^{n_1}} - (n_2+1)\frac{(n_1+n_2)!}{n_1!\,2^{n_1}}\right\}$$

$$= (n_2+2)\sum_{r=0}^{n_1} r\frac{(r+n_2)!}{r!\,2^r} + (n_2+1)\sum_{r=0}^{n_1}\frac{(r+n_2)!}{r!\,2^r} - (n_1+1)\frac{(n_1+n_2+1)!}{n_1!\,2^{n_1}}$$

$$= (n_2+2)\left\{(n_2+1)X - Y\right\} + (n_2+1)X - (n_1+1)Y$$

$$= (n_2+3)\,(n_2+1)X - (n_1+n_2+3)Y.$$

The mean value of the squared activity is now found to be

$$E[a^2] = (n_1+1)(n_1+2) - (2n_1+3)\frac{(n_2+1)X-Y}{X} + \frac{(n_2+3)(n_2+1)X-(n_1+n_2+3)Y}{X}$$

$$= (n_1-n_2)^2 + (n_1+n_2+3) + (n_1-n_2)\frac{Y}{X}.$$

The square of the standard deviation is then

$$D^2[a] = E[a^2] - E^2[a]$$

$$= (n_1-n_2)^2 + (n_1+n_2+3) + (n_1-n_2)\frac{Y}{X}$$

$$- (n_1-n_2+\frac{Y}{X})^2$$

$$= n_1+n_2+3 - (n_1-n_2)\cdot\frac{Y}{X} - (\frac{Y}{X})^2.$$

As shown above,

$$\frac{Y}{X} \rightarrow 0 \quad \text{for} \quad n_1 \rightarrow \infty \quad \text{and} \quad n_2 \text{ constant,}$$

and thus

$$D^2 [a] \rightarrow n_1 + n_2 + 3$$

under the same conditions.

Appendix 3

Summation of $\sum_{r=0}^{n_1} \frac{(n_2+r)!}{r!} (\frac{1}{2})^r$

The sum occurring in the denominator in the formula for the mean value and the standard deviation is a little unhandy for direct use when the observed count numbers n_1 and n_2 become larger. In order to find an approximative expression we convert the series into a function $f(n_1, n_2, x)$ defined by

$$f(n_1, n_2, x) = \sum_{r=0}^{n_1} \frac{(r+n_2)!}{r!} x^r.$$

This function is differentiable with the differential quotient

$$\frac{\partial}{\partial x} f(n_1, n_2, x) = \sum_{r=0}^{n_1} \frac{(r+n_2)!}{r!} r\, x^{r-1}$$

$$= \sum_{r-1=0}^{n_1-1} \frac{(r-1+n_2+1)!}{(r-1)!} x^{r-1}$$

$$= f(n_1-1,\ n_2+1,\ x).$$

Further, we find that

$$f(n_1, n_2+1, x) = \sum_{r=0}^{n_1} \frac{(r+n_2+1)!}{r!} x^r$$

$$= x \sum_{r=0}^{n_1} \frac{(r+n_2)!}{r!} r\, x^{r-1} + (n_2+1) \sum_{r=0}^{n_1} \frac{(r+n_2)!}{r!} x^r$$

$$= x \cdot f(n_1-1, n_2+1, x) + (n_2+1)\, f(n_1, n_2, x)$$

and

$$f(n_1, n_2+1, x) = \sum_{r=0}^{n_1} \frac{(r+n_2+1)!}{r!} x^r$$

$$= \sum_{r=0}^{n_1} \frac{(r+n_2+1)!}{r!} x^r + \frac{(n_1+n_2+1)!}{n_1!} x^{n_1}$$

$$= f(n_1-1, n_2+1, x) + \frac{(n_1+n_2+1)!}{n_1!} x^{n_1} .$$

A combination of these two equations gives

$$f(n_1-1, n_2+1, x) = \frac{1}{1-x} \quad (n_2+1)\, f(n_1, n_2, x) - \frac{(n_1+n_2+1)!}{n_1!} x^{n_1}$$

$$= \frac{\partial}{\partial x} f(n_1, n_2, x).$$

This is a linear first-order differential equation:

$$\frac{\partial}{\partial x} f(n_1, n_2, x) - \frac{n_2+1}{1-x} f(n_1, n_2, x) = - \frac{(n_1+n_2+1)!}{n_1!} \frac{x^{n_1}}{1-x},$$

of the form

$$\frac{dy}{dx} + f(x)y = g(x),$$

where

$$f(x) = - \frac{n_2+1}{1-x}$$

and [illegible] above,

$$g(x) = -\frac{(n_1+n_2+1)!}{n_1!} \frac{x^{n_1}}{1-x}.$$

The solution to such an equation is given by

$$y = e^{-\int f(x)dx} \cdot \left\{ \int g(x)e^{\int f(x)dx} dx + C \right\},$$

where

$$\int_o^x f(x)dx = \int_o^x -\frac{n_2+1}{1-x} dx$$

$$= \ln(1-x)^{n_2+1}$$

and

$$\int_o^x g(x)e^{\int_o^x f(x)dx} dx = \int_o^x -\frac{(n_1+n_2+1)!}{n_1!} \frac{x^{n_1}}{1-x} \cdot (1-x)^{n_2+1} dx$$

$$= -\frac{(n_1+n_2+1)!}{n_1!} \int_o^x x^{n_1}(1-x)^{n_2} dx.$$

By insertion we obtain

$$f(n_1, n_2, x) = \frac{1}{(1-x)^{n_2+1}} \left\{ -\frac{(n_1+n_2+1)!}{n_1!} \int_o^x x^{n_1}(1-x)^{n_2} dx + C \right\}$$

$$\rightarrow C \quad \text{as} \quad x \rightarrow 0.$$

It is directly seen that

$$f(n_1, n_2, x) = \sum_{r=0}^{n_1} \frac{(r+n_2)!}{r!} x^r$$

$$\rightarrow n_2! \quad \text{as} \quad x \rightarrow 0,$$

and this gives

$$C = n_2! \;.$$

The solution is thus

$$f(n_1, n_2, x) = \frac{1}{(1-x)^{n_2+1}} \left\{ - \frac{(n_1+n_2+1)!}{n_1!} \int_0^x x^{n_1}(1-x)^{n_2}dx + n_2! \right\} ,$$

and, taking $x = \frac{1}{2}$, we find

$$\sum_{r=0}^{n_1} \frac{(r+n_2)!}{r!} \left(\frac{1}{2}\right)^r = 2^{n_2+1} \left\{ n_2! - \frac{(n_1+n_2+1)!}{n_1!} \int_0^{\frac{1}{2}} x^{n_1}(1-x)^{n_2}dx \right\} .$$

By using the formula [6)]

$$\int_0^1 x^{n_1}(1-x)^{n_2}dx = \int_0^{\frac{1}{2}} x^{n_1}(1-x)^{n_2}dx + \int_{\frac{1}{2}}^1 x^{n_1}(1-x)^{n_2}dx$$

$$= \int_0^{\frac{1}{2}} x^{n_1}(1-x)^{n_2}dx + \int_0^{\frac{1}{2}} x^{n_2}(1-x)^{n_1}dx$$

$$= \frac{n_1!\, n_2!}{(n_1+n_2+1)!}$$

we obtain

$$\sum_{r=0}^{n_1} \frac{(r+n_2)!}{r!} \left(\frac{1}{2}\right)^r = 2^{n_2+1} \frac{(n_1+n_2+1)!}{n_1!} \int_0^{\frac{1}{2}} x^{n_2}(1-x)^{n_1}dx.$$

Table 1

Table 1 gives the mean values E [a] and the relative standard deviations D [a] /E [a] for

$$1 \leqslant n_1 \leqslant 51$$

and

$$1 \leqslant n_2 \leqslant 51.$$

The table has been calculated on the basis of the exact formulae.

Table 1

n1	n2 = 1		n2 = 2		n2 = 3	
n1	E[a]	D[a]/E[a]	E[a]	D[a]/E[a]	E[a]	D[a]/E[a]
1	1.500	$8.819_{10}-1$	1.400	$9.147_{10}-1$	1.333	$9.354_{10}-1$
2	2.091	$7.886_{10}-1$	1.875	$8.406_{10}-1$	1.727	$8.760_{10}-1$
3	2.769	$7.104_{10}-1$	2.429	$7.737_{10}-1$	2.188	$8.197_{10}-1$
4	3.526	$6.432_{10}-1$	3.061	$7.124_{10}-1$	2.718	$7.658_{10}-1$
5	4.350	$5.851_{10}-1$	3.767	$6.562_{10}-1$	3.319	$7.142_{10}-1$
6	5.227	$5.349_{10}-1$	4.541	$6.049_{10}-1$	3.991	$6.652_{10}-1$
7	6.143	$4.915_{10}-1$	5.372	$5.585_{10}-1$	4.727	$6.191_{10}-1$
8	7.089	$4.543_{10}-1$	6.250	$5.169_{10}-1$	5.521	$5.762_{10}-1$
9	8.054	$4.222_{10}-1$	7.164	$4.799_{10}-1$	6.366	$5.366_{10}-1$
10	9.032	$3.947_{10}-1$	8.106	$4.472_{10}-1$	7.252	$5.006_{10}-1$
11	$1.002_{10}\ 1$	$3.709_{10}-1$	9.067	$4.184_{10}-1$	8.170	$4.679_{10}-1$
12	$1.101_{10}\ 1$	$3.503_{10}-1$	$1.004_{10}\ 1$	$3.931_{10}-1$	9.112	$4.387_{10}-1$
13	$1.201_{10}\ 1$	$3.324_{10}-1$	$1.103_{10}\ 1$	$3.708_{10}-1$	$1.007_{10}\ 1$	$4.125_{10}-1$
14	$1.300_{10}\ 1$	$3.166_{10}-1$	$1.202_{10}\ 1$	$3.513_{10}-1$	$1.105_{10}\ 1$	$3.892_{10}-1$
15	$1.400_{10}\ 1$	$3.028_{10}-1$	$1.301_{10}\ 1$	$3.340_{10}-1$	$1.203_{10}\ 1$	$3.684_{10}-1$
16	$1.500_{10}\ 1$	$2.904_{10}-1$	$1.401_{10}\ 1$	$3.187_{10}-1$	$1.302_{10}\ 1$	$3.500_{10}-1$
17	$1.600_{10}\ 1$	$2.794_{10}-1$	$1.500_{10}\ 1$	$3.051_{10}-1$	$1.401_{10}\ 1$	$3.335_{10}-1$
18	$1.700_{10}\ 1$	$2.695_{10}-1$	$1.600_{10}\ 1$	$2.929_{10}-1$	$1.501_{10}\ 1$	$3.188_{10}-1$
19	$1.800_{10}\ 1$	$2.606_{10}-1$	$1.700_{10}\ 1$	$2.820_{10}-1$	$1.600_{10}\ 1$	$3.057_{10}-1$
20	$1.900_{10}\ 1$	$2.524_{10}-1$	$1.800_{10}\ 1$	$2.721_{10}-1$	$1.700_{10}\ 1$	$2.938_{10}-1$
21	$2.000_{10}\ 1$	$2.449_{10}-1$	$1.900_{10}\ 1$	$2.631_{10}-1$	$1.800_{10}\ 1$	$2.831_{10}-1$
22	$2.100_{10}\ 1$	$2.381_{10}-1$	$2.000_{10}\ 1$	$2.549_{10}-1$	$1.900_{10}\ 1$	$2.734_{10}-1$
23	$2.200_{10}\ 1$	$2.318_{10}-1$	$2.100_{10}\ 1$	$2.474_{10}-1$	$2.000_{10}\ 1$	$2.645_{10}-1$
24	$2.300_{10}\ 1$	$2.259_{10}-1$	$2.200_{10}\ 1$	$2.405_{10}-1$	$2.100_{10}\ 1$	$2.564_{10}-1$
25	$2.400_{10}\ 1$	$2.205_{10}-1$	$2.300_{10}\ 1$	$2.341_{10}-1$	$2.200_{10}\ 1$	$2.489_{10}-1$
26	$2.500_{10}\ 1$	$2.154_{10}-1$	$2.400_{10}\ 1$	$2.282_{10}-1$	$2.300_{10}\ 1$	$2.421_{10}-1$
27	$2.600_{10}\ 1$	$2.107_{10}-1$	$2.500_{10}\ 1$	$2.227_{10}-1$	$2.400_{10}\ 1$	$2.357_{10}-1$
28	$2.700_{10}\ 1$	$2.062_{10}-1$	$2.600_{10}\ 1$	$2.176_{10}-1$	$2.500_{10}\ 1$	$2.298_{10}-1$
29	$2.800_{10}\ 1$	$2.020_{10}-1$	$2.700_{10}\ 1$	$2.128_{10}-1$	$2.600_{10}\ 1$	$2.243_{10}-1$
30	$2.900_{10}\ 1$	$1.981_{10}-1$	$2.800_{10}\ 1$	$2.082_{10}-1$	$2.700_{10}\ 1$	$2.191_{10}-1$
31	$3.000_{10}\ 1$	$1.944_{10}-1$	$2.900_{10}\ 1$	$2.040_{10}-1$	$2.800_{10}\ 1$	$2.143_{10}-1$
32	$3.100_{10}\ 1$	$1.908_{10}-1$	$3.000_{10}\ 1$	$2.000_{10}-1$	$2.900_{10}\ 1$	$2.098_{10}-1$
33	$3.200_{10}\ 1$	$1.875_{10}-1$	$3.100_{10}\ 1$	$1.962_{10}-1$	$3.000_{10}\ 1$	$2.055_{10}-1$
34	$3.300_{10}\ 1$	$1.843_{10}-1$	$3.200_{10}\ 1$	$1.926_{10}-1$	$3.100_{10}\ 1$	$2.015_{10}-1$
35	$3.400_{10}\ 1$	$1.813_{10}-1$	$3.300_{10}\ 1$	$1.892_{10}-1$	$3.200_{10}\ 1$	$1.976_{10}-1$
36	$3.500_{10}\ 1$	$1.784_{10}-1$	$3.400_{10}\ 1$	$1.860_{10}-1$	$3.300_{10}\ 1$	$1.940_{10}-1$
37	$3.600_{10}\ 1$	$1.757_{10}-1$	$3.500_{10}\ 1$	$1.829_{10}-1$	$3.400_{10}\ 1$	$1.906_{10}-1$
38	$3.700_{10}\ 1$	$1.731_{10}-1$	$3.600_{10}\ 1$	$1.800_{10}-1$	$3.500_{10}\ 1$	$1.874_{10}-1$
39	$3.800_{10}\ 1$	$1.705_{10}-1$	$3.700_{10}\ 1$	$1.772_{10}-1$	$3.600_{10}\ 1$	$1.843_{10}-1$
40	$3.900_{10}\ 1$	$1.681_{10}-1$	$3.800_{10}\ 1$	$1.746_{10}-1$	$3.700_{10}\ 1$	$1.813_{10}-1$
41	$4.000_{10}\ 1$	$1.658_{10}-1$	$3.900_{10}\ 1$	$1.720_{10}-1$	$3.800_{10}\ 1$	$1.785_{10}-1$
42	$4.100_{10}\ 1$	$1.636_{10}-1$	$4.000_{10}\ 1$	$1.696_{10}-1$	$3.900_{10}\ 1$	$1.758_{10}-1$
43	$4.200_{10}\ 1$	$1.615_{10}-1$	$4.100_{10}\ 1$	$1.672_{10}-1$	$4.000_{10}\ 1$	$1.732_{10}-1$
44	$4.300_{10}\ 1$	$1.594_{10}-1$	$4.200_{10}\ 1$	$1.650_{10}-1$	$4.100_{10}\ 1$	$1.707_{10}-1$
45	$4.400_{10}\ 1$	$1.575_{10}-1$	$4.300_{10}\ 1$	$1.628_{10}-1$	$4.200_{10}\ 1$	$1.684_{10}-1$
46	$4.500_{10}\ 1$	$1.556_{10}-1$	$4.400_{10}\ 1$	$1.607_{10}-1$	$4.300_{10}\ 1$	$1.661_{10}-1$
47	$4.600_{10}\ 1$	$1.537_{10}-1$	$4.500_{10}\ 1$	$1.587_{10}-1$	$4.400_{10}\ 1$	$1.639_{10}-1$
48	$4.700_{10}\ 1$	$1.519_{10}-1$	$4.600_{10}\ 1$	$1.568_{10}-1$	$4.500_{10}\ 1$	$1.618_{10}-1$
49	$4.800_{10}\ 1$	$1.502_{10}-1$	$4.700_{10}\ 1$	$1.549_{10}-1$	$4.600_{10}\ 1$	$1.597_{10}-1$
50	$4.900_{10}\ 1$	$1.486_{10}-1$	$4.800_{10}\ 1$	$1.531_{10}-1$	$4.700_{10}\ 1$	$1.578_{10}-1$
51	$5.000_{10}\ 1$	$1.470_{10}-1$	$4.900_{10}\ 1$	$1.514_{10}-1$	$4.800_{10}\ 1$	$1.559_{10}-1$

n1	n2 = 4: E[a]	n2 = 4: D[a]/E[a]	n2 = 5: E[a]	n2 = 5: D[a]/E[a]	n2 = 6: E[a]	n2 = 6: D[a]/E[a]
1	1.286	$9.493_{10}-1$	1.250	$9.592_{10}-1$	1.222	$9.664_{10}-1$
2	1.621	$9.009_{10}-1$	1.541	$9.192_{10}-1$	1.478	$9.329_{10}-1$
3	2.011	$8.536_{10}-1$	1.877	$8.792_{10}-1$	1.773	$8.989_{10}-1$
4	2.461	$8.070_{10}-1$	2.264	$8.390_{10}-1$	2.110	$8.642_{10}-1$
5	2.975	$7.610_{10}-1$	2.707	$7.985_{10}-1$	2.496	$8.286_{10}-1$
6	3.555	$7.159_{10}-1$	3.209	$7.578_{10}-1$	2.933	$7.924_{10}-1$
7	4.199	$6.720_{10}-1$	3.772	$7.173_{10}-1$	3.425	$7.556_{10}-1$
8	4.906	$6.299_{10}-1$	4.396	$6.774_{10}-1$	3.974	$7.186_{10}-1$
9	5.671	$5.899_{10}-1$	5.079	$6.385_{10}-1$	4.581	$6.818_{10}-1$
10	6.487	$5.524_{10}-1$	5.819	$6.010_{10}-1$	5.246	$6.455_{10}-1$
11	7.347	$5.175_{10}-1$	6.610	$5.653_{10}-1$	5.965	$6.102_{10}-1$
12	8.242	$4.854_{10}-1$	7.446	$5.317_{10}-1$	6.734	$5.762_{10}-1$
13	9.166	$4.561_{10}-1$	8.321	$5.003_{10}-1$	7.549	$5.438_{10}-1$
14	$1.011_{10}\ 1$	$4.296_{10}-1$	9.226	$4.713_{10}-1$	8.404	$5.133_{10}-1$
15	$1.107_{10}\ 1$	$4.056_{10}-1$	$1.016_{10}\ 1$	$4.447_{10}-1$	9.292	$4.847_{10}-1$
16	$1.205_{10}\ 1$	$3.841_{10}-1$	$1.111_{10}\ 1$	$4.204_{10}-1$	$1.021_{10}\ 1$	$4.582_{10}-1$
17	$1.303_{10}\ 1$	$3.647_{10}-1$	$1.207_{10}\ 1$	$3.983_{10}-1$	$1.115_{10}\ 1$	$4.337_{10}-1$
18	$1.402_{10}\ 1$	$3.474_{10}-1$	$1.305_{10}\ 1$	$3.783_{10}-1$	$1.210_{10}\ 1$	$4.113_{10}-1$
19	$1.501_{10}\ 1$	$3.318_{10}-1$	$1.403_{10}\ 1$	$3.603_{10}-1$	$1.307_{10}\ 1$	$3.909_{10}-1$
20	$1.601_{10}\ 1$	$3.178_{10}-1$	$1.502_{10}\ 1$	$3.439_{10}-1$	$1.405_{10}\ 1$	$3.723_{10}-1$
21	$1.700_{10}\ 1$	$3.051_{10}-1$	$1.601_{10}\ 1$	$3.292_{10}-1$	$1.503_{10}\ 1$	$3.554_{10}-1$
22	$1.800_{10}\ 1$	$2.936_{10}-1$	$1.701_{10}\ 1$	$3.158_{10}-1$	$1.602_{10}\ 1$	$3.400_{10}-1$
23	$1.900_{10}\ 1$	$2.832_{10}-1$	$1.801_{10}\ 1$	$3.037_{10}-1$	$1.701_{10}\ 1$	$3.261_{10}-1$
24	$2.000_{10}\ 1$	$2.737_{10}-1$	$1.900_{10}\ 1$	$2.927_{10}-1$	$1.801_{10}\ 1$	$3.134_{10}-1$
25	$2.100_{10}\ 1$	$2.651_{10}-1$	$2.000_{10}\ 1$	$2.826_{10}-1$	$1.901_{10}\ 1$	$3.018_{10}-1$
26	$2.200_{10}\ 1$	$2.571_{10}-1$	$2.100_{10}\ 1$	$2.734_{10}-1$	$2.000_{10}\ 1$	$2.912_{10}-1$
27	$2.300_{10}\ 1$	$2.497_{10}-1$	$2.200_{10}\ 1$	$2.650_{10}-1$	$2.100_{10}\ 1$	$2.815_{10}-1$
28	$2.400_{10}\ 1$	$2.429_{10}-1$	$2.300_{10}\ 1$	$2.572_{10}-1$	$2.200_{10}\ 1$	$2.726_{10}-1$
29	$2.500_{10}\ 1$	$2.366_{10}-1$	$2.400_{10}\ 1$	$2.500_{10}-1$	$2.300_{10}\ 1$	$2.644_{10}-1$
30	$2.600_{10}\ 1$	$2.308_{10}-1$	$2.500_{10}\ 1$	$2.433_{10}-1$	$2.400_{10}\ 1$	$2.568_{10}-1$
31	$2.700_{10}\ 1$	$2.253_{10}-1$	$2.600_{10}\ 1$	$2.371_{10}-1$	$2.500_{10}\ 1$	$2.498_{10}-1$
32	$2.800_{10}\ 1$	$2.202_{10}-1$	$2.700_{10}\ 1$	$2.313_{10}-1$	$2.600_{10}\ 1$	$2.432_{10}-1$
33	$2.900_{10}\ 1$	$2.153_{10}-1$	$2.800_{10}\ 1$	$2.259_{10}-1$	$2.700_{10}\ 1$	$2.371_{10}-1$
34	$3.000_{10}\ 1$	$2.108_{10}-1$	$2.900_{10}\ 1$	$2.208_{10}-1$	$2.800_{10}\ 1$	$2.314_{10}-1$
35	$3.100_{10}\ 1$	$2.066_{10}-1$	$3.000_{10}\ 1$	$2.160_{10}-1$	$2.900_{10}\ 1$	$2.261_{10}-1$
36	$3.200_{10}\ 1$	$2.025_{10}-1$	$3.100_{10}\ 1$	$2.115_{10}-1$	$3.000_{10}\ 1$	$2.211_{10}-1$
37	$3.300_{10}\ 1$	$1.987_{10}-1$	$3.200_{10}\ 1$	$2.073_{10}-1$	$3.100_{10}\ 1$	$2.164_{10}-1$
38	$3.400_{10}\ 1$	$1.951_{10}-1$	$3.300_{10}\ 1$	$2.033_{10}-1$	$3.200_{10}\ 1$	$2.119_{10}-1$
39	$3.500_{10}\ 1$	$1.917_{10}-1$	$3.400_{10}\ 1$	$1.995_{10}-1$	$3.300_{10}\ 1$	$2.077_{10}-1$
40	$3.600_{10}\ 1$	$1.884_{10}-1$	$3.500_{10}\ 1$	$1.959_{10}-1$	$3.400_{10}\ 1$	$2.038_{10}-1$
41	$3.700_{10}\ 1$	$1.853_{10}-1$	$3.600_{10}\ 1$	$1.925_{10}-1$	$3.500_{10}\ 1$	$2.000_{10}-1$
42	$3.800_{10}\ 1$	$1.823_{10}-1$	$3.700_{10}\ 1$	$1.892_{10}-1$	$3.600_{10}\ 1$	$1.964_{10}-1$
43	$3.900_{10}\ 1$	$1.795_{10}-1$	$3.800_{10}\ 1$	$1.861_{10}-1$	$3.700_{10}\ 1$	$1.930_{10}-1$
44	$4.000_{10}\ 1$	$1.768_{10}-1$	$3.900_{10}\ 1$	$1.831_{10}-1$	$3.800_{10}\ 1$	$1.898_{10}-1$
45	$4.100_{10}\ 1$	$1.742_{10}-1$	$4.000_{10}\ 1$	$1.803_{10}-1$	$3.900_{10}\ 1$	$1.867_{10}-1$
46	$4.200_{10}\ 1$	$1.717_{10}-1$	$4.100_{10}\ 1$	$1.776_{10}-1$	$4.000_{10}\ 1$	$1.837_{10}-1$
47	$4.300_{10}\ 1$	$1.693_{10}-1$	$4.200_{10}\ 1$	$1.750_{10}-1$	$4.100_{10}\ 1$	$1.809_{10}-1$
48	$4.400_{10}\ 1$	$1.670_{10}-1$	$4.300_{10}\ 1$	$1.725_{10}-1$	$4.200_{10}\ 1$	$1.782_{10}-1$
49	$4.500_{10}\ 1$	$1.648_{10}-1$	$4.400_{10}\ 1$	$1.701_{10}-1$	$4.300_{10}\ 1$	$1.756_{10}-1$
50	$4.600_{10}\ 1$	$1.627_{10}-1$	$4.500_{10}\ 1$	$1.678_{10}-1$	$4.400_{10}\ 1$	$1.731_{10}-1$
51	$4.700_{10}\ 1$	$1.606_{10}-1$	$4.600_{10}\ 1$	$1.656_{10}-1$	$4.500_{10}\ 1$	$1.707_{10}-1$

	n2 = 7		n2 = 8		n2 = 9	
n1	E[a]	D[a]/E[a]	E[a]	D[a]/E[a]	E[a]	D[a]/E[a]
1	1.200	9.718$_{10}$-1	1.182	9.760$_{10}$-1	1.167	9.794$_{10}$-1
2	1.429	9.434$_{10}$-1	1.388	9.517$_{10}$-1	1.354	9.583$_{10}$-1
3	1.690	9.143$_{10}$-1	1.622	9.265$_{10}$-1	1.566	9.363$_{10}$-1
4	1.987	8.842$_{10}$-1	1.887	9.003$_{10}$-1	1.805	9.134$_{10}$-1
5	2.326	8.531$_{10}$-1	2.188	8.730$_{10}$-1	2.074	8.893$_{10}$-1
6	2.710	8.209$_{10}$-1	2.528	8.445$_{10}$-1	2.377	8.641$_{10}$-1
7	3.142	7.878$_{10}$-1	2.910	8.149$_{10}$-1	2.717	8.376$_{10}$-1
8	3.626	7.540$_{10}$-1	3.338	7.842$_{10}$-1	3.099	8.100$_{10}$-1
9	4.165	7.198$_{10}$-1	3.816	7.528$_{10}$-1	3.524	7.814$_{10}$-1
10	4.758	6.855$_{10}$-1	4.345	7.208$_{10}$-1	3.996	7.519$_{10}$-1
11	5.406	6.514$_{10}$-1	4.927	6.886$_{10}$-1	4.517	7.218$_{10}$-1
12	6.107	6.180$_{10}$-1	5.560	6.564$_{10}$-1	5.088	6.913$_{10}$-1
13	6.857	5.855$_{10}$-1	6.246	6.246$_{10}$-1	5.710	6.608$_{10}$-1
14	7.654	5.543$_{10}$-1	6.980	5.936$_{10}$-1	6.381	6.304$_{10}$-1
15	8.490	5.246$_{10}$-1	7.759	5.634$_{10}$-1	7.100	6.006$_{10}$-1
16	9.362	4.965$_{10}$-1	8.579	5.345$_{10}$-1	7.864	5.715$_{10}$-1
17	1.026$_{10}$ 1	4.703$_{10}$-1	9.435	5.071$_{10}$-1	8.669	5.434$_{10}$-1
18	1.119$_{10}$ 1	4.458$_{10}$-1	1.032$_{10}$ 1	4.812$_{10}$-1	9.511	5.166$_{10}$-1
19	1.213$_{10}$ 1	4.233$_{10}$-1	1.124$_{10}$ 1	4.569$_{10}$-1	1.038$_{10}$ 1	4.910$_{10}$-1
20	1.309$_{10}$ 1	4.025$_{10}$-1	1.217$_{10}$ 1	4.343$_{10}$-1	1.129$_{10}$ 1	4.670$_{10}$-1
21	1.406$_{10}$ 1	3.835$_{10}$-1	1.312$_{10}$ 1	4.133$_{10}$-1	1.221$_{10}$ 1	4.444$_{10}$-1
22	1.504$_{10}$ 1	3.662$_{10}$-1	1.408$_{10}$ 1	3.940$_{10}$-1	1.315$_{10}$ 1	4.234$_{10}$-1
23	1.603$_{10}$ 1	3.503$_{10}$-1	1.506$_{10}$ 1	3.763$_{10}$-1	1.411$_{10}$ 1	4.039$_{10}$-1
24	1.702$_{10}$ 1	3.358$_{10}$-1	1.604$_{10}$ 1	3.600$_{10}$-1	1.508$_{10}$ 1	3.859$_{10}$-1
25	1.801$_{10}$ 1	3.226$_{10}$-1	1.703$_{10}$ 1	3.451$_{10}$-1	1.605$_{10}$ 1	3.693$_{10}$-1
26	1.901$_{10}$ 1	3.105$_{10}$-1	1.802$_{10}$ 1	3.314$_{10}$-1	1.704$_{10}$ 1	3.540$_{10}$-1
27	2.001$_{10}$ 1	2.995$_{10}$-1	1.901$_{10}$ 1	3.189$_{10}$-1	1.803$_{10}$ 1	3.399$_{10}$-1
28	2.100$_{10}$ 1	2.893$_{10}$-1	2.001$_{10}$ 1	3.074$_{10}$-1	1.902$_{10}$ 1	3.270$_{10}$-1
29	2.200$_{10}$ 1	2.800$_{10}$-1	2.101$_{10}$ 1	2.969$_{10}$-1	2.001$_{10}$ 1	3.151$_{10}$-1
30	2.300$_{10}$ 1	2.714$_{10}$-1	2.200$_{10}$ 1	2.872$_{10}$-1	2.101$_{10}$ 1	3.042$_{10}$-1
31	2.400$_{10}$ 1	2.634$_{10}$-1	2.300$_{10}$ 1	2.782$_{10}$-1	2.201$_{10}$ 1	2.941$_{10}$-1
32	2.500$_{10}$ 1	2.561$_{10}$-1	2.400$_{10}$ 1	2.699$_{10}$-1	2.300$_{10}$ 1	2.848$_{10}$-1
33	2.600$_{10}$ 1	2.492$_{10}$-1	2.500$_{10}$ 1	2.622$_{10}$-1	2.400$_{10}$ 1	2.762$_{10}$-1
34	2.700$_{10}$ 1	2.428$_{10}$-1	2.600$_{10}$ 1	2.551$_{10}$-1	2.500$_{10}$ 1	2.682$_{10}$-1
35	2.800$_{10}$ 1	2.369$_{10}$-1	2.700$_{10}$ 1	2.484$_{10}$-1	2.600$_{10}$ 1	2.608$_{10}$-1
36	2.900$_{10}$ 1	2.313$_{10}$-1	2.800$_{10}$ 1	2.422$_{10}$-1	2.700$_{10}$ 1	2.539$_{10}$-1
37	3.000$_{10}$ 1	2.261$_{10}$-1	2.900$_{10}$ 1	2.364$_{10}$-1	2.800$_{10}$ 1	2.474$_{10}$-1
38	3.100$_{10}$ 1	2.211$_{10}$-1	3.000$_{10}$ 1	2.309$_{10}$-1	2.900$_{10}$ 1	2.414$_{10}$-1
39	3.200$_{10}$ 1	2.165$_{10}$-1	3.100$_{10}$ 1	2.258$_{10}$-1	3.000$_{10}$ 1	2.357$_{10}$-1
40	3.300$_{10}$ 1	2.121$_{10}$-1	3.200$_{10}$ 1	2.210$_{10}$-1	3.100$_{10}$ 1	2.304$_{10}$-1
41	3.400$_{10}$ 1	2.080$_{10}$-1	3.300$_{10}$ 1	2.164$_{10}$-1	3.200$_{10}$ 1	2.253$_{10}$-1
42	3.500$_{10}$ 1	2.040$_{10}$-1	3.400$_{10}$ 1	2.121$_{10}$-1	3.300$_{10}$ 1	2.206$_{10}$-1
43	3.600$_{10}$ 1	2.003$_{10}$-1	3.500$_{10}$ 1	2.080$_{10}$-1	3.400$_{10}$ 1	2.161$_{10}$-1
44	3.700$_{10}$ 1	1.968$_{10}$-1	3.600$_{10}$ 1	2.041$_{10}$-1	3.500$_{10}$ 1	2.119$_{10}$-1
45	3.800$_{10}$ 1	1.934$_{10}$-1	3.700$_{10}$ 1	2.004$_{10}$-1	3.600$_{10}$ 1	2.079$_{10}$-1
46	3.900$_{10}$ 1	1.902$_{10}$-1	3.800$_{10}$ 1	1.969$_{10}$-1	3.700$_{10}$ 1	2.040$_{10}$-1
47	4.000$_{10}$ 1	1.871$_{10}$-1	3.900$_{10}$ 1	1.936$_{10}$-1	3.800$_{10}$ 1	2.004$_{10}$-1
48	4.100$_{10}$ 1	1.841$_{10}$-1	4.000$_{10}$ 1	1.904$_{10}$-1	3.900$_{10}$ 1	1.970$_{10}$-1
49	4.200$_{10}$ 1	1.813$_{10}$-1	4.100$_{10}$ 1	1.873$_{10}$-1	4.000$_{10}$ 1	1.936$_{10}$-1
50	4.300$_{10}$ 1	1.786$_{10}$-1	4.200$_{10}$ 1	1.844$_{10}$-1	4.100$_{10}$ 1	1.905$_{10}$-1
51	4.400$_{10}$ 1	1.760$_{10}$-1	4.300$_{10}$ 1	1.816$_{10}$-1	4.200$_{10}$ 1	1.875$_{10}$-1

	n2 = 10		n2 = 11		n2 = 12	
n1	E[a]	D[a]/E[a]	E[a]	D[a]/E[a]	E[a]	D[a]/E[a]
1	1.154	9.821$_{10}$-1	1.143	9.843$_{10}$-1	1.133	9.861$_{10}$-1
2	1.326	9.636$_{10}$-1	1.302	9.680$_{10}$-1	1.281	9.716$_{10}$-1
3	1.519	9.444$_{10}$-1	1.479	9.510$_{10}$-1	1.445	9.565$_{10}$-1
4	1.736	9.242$_{10}$-1	1.677	9.331$_{10}$-1	1.627	9.406$_{10}$-1
5	1.979	9.029$_{10}$-1	1.898	9.142$_{10}$-1	1.829	9.238$_{10}$-1
6	2.251	8.805$_{10}$-1	2.145	8.942$_{10}$-1	2.054	9.059$_{10}$-1
7	2.556	8.568$_{10}$-1	2.420	8.731$_{10}$-1	2.304	8.869$_{10}$-1
8	2.897	8.320$_{10}$-1	2.727	8.508$_{10}$-1	2.582	8.669$_{10}$-1
9	3.278	8.060$_{10}$-1	3.069	8.273$_{10}$-1	2.890	8.456$_{10}$-1
10	3.700	7.790$_{10}$-1	3.448	8.027$_{10}$-1	3.233	8.233$_{10}$-1
11	4.167	7.512$_{10}$-1	3.868	7.771$_{10}$-1	3.612	7.998$_{10}$-1
12	4.682	7.227$_{10}$-1	4.331	7.506$_{10}$-1	4.030	7.754$_{10}$-1
13	5.244	6.937$_{10}$-1	4.839	7.235$_{10}$-1	4.489	7.502$_{10}$-1
14	5.855	6.646$_{10}$-1	5.394	6.959$_{10}$-1	4.991	7.242$_{10}$-1
15	6.514	6.355$_{10}$-1	5.995	6.680$_{10}$-1	5.539	6.978$_{10}$-1
16	7.220	6.068$_{10}$-1	6.643	6.401$_{10}$-1	6.132	6.710$_{10}$-1
17	7.970	5.787$_{10}$-1	7.337	6.124$_{10}$-1	6.770	6.442$_{10}$-1
18	8.761	5.514$_{10}$-1	8.074	5.852$_{10}$-1	7.453	6.174$_{10}$-1
19	9.589	5.251$_{10}$-1	8.852	5.586$_{10}$-1	8.178	5.910$_{10}$-1
20	1.045$_{10}$ 1	5.000$_{10}$-1	9.668	5.329$_{10}$-1	8.944	5.652$_{10}$-1
21	1.134$_{10}$ 1	4.762$_{10}$-1	1.052$_{10}$ 1	5.083$_{10}$-1	9.748	5.401$_{10}$-1
22	1.225$_{10}$ 1	4.538$_{10}$-1	1.139$_{10}$ 1	4.847$_{10}$-1	1.058$_{10}$ 1	5.158$_{10}$-1
23	1.319$_{10}$ 1	4.327$_{10}$-1	1.230$_{10}$ 1	4.624$_{10}$-1	1.145$_{10}$ 1	4.926$_{10}$-1
24	1.413$_{10}$ 1	4.131$_{10}$-1	1.322$_{10}$ 1	4.415$_{10}$-1	1.235$_{10}$ 1	4.705$_{10}$-1
25	1.510$_{10}$ 1	3.949$_{10}$-1	1.416$_{10}$ 1	4.218$_{10}$-1	1.326$_{10}$ 1	4.496$_{10}$-1
26	1.607$_{10}$ 1	3.780$_{10}$-1	1.512$_{10}$ 1	4.034$_{10}$-1	1.419$_{10}$ 1	4.299$_{10}$-1
27	1.705$_{10}$ 1	3.624$_{10}$-1	1.609$_{10}$ 1	3.863$_{10}$-1	1.514$_{10}$ 1	4.115$_{10}$-1
28	1.803$_{10}$ 1	3.480$_{10}$-1	1.706$_{10}$ 1	3.705$_{10}$-1	1.610$_{10}$ 1	3.942$_{10}$-1
29	1.902$_{10}$ 1	3.348$_{10}$-1	1.804$_{10}$ 1	3.558$_{10}$-1	1.708$_{10}$ 1	3.782$_{10}$-1
30	2.002$_{10}$ 1	3.226$_{10}$-1	1.903$_{10}$ 1	3.423$_{10}$-1	1.805$_{10}$ 1	3.633$_{10}$-1
31	2.101$_{10}$ 1	3.113$_{10}$-1	2.002$_{10}$ 1	3.297$_{10}$-1	1.904$_{10}$ 1	3.495$_{10}$-1
32	2.201$_{10}$ 1	3.009$_{10}$-1	2.101$_{10}$ 1	3.181$_{10}$-1	2.003$_{10}$ 1	3.367$_{10}$-1
33	2.300$_{10}$ 1	2.912$_{10}$-1	2.201$_{10}$ 1	3.074$_{10}$-1	2.102$_{10}$ 1	3.248$_{10}$-1
34	2.400$_{10}$ 1	2.823$_{10}$-1	2.301$_{10}$ 1	2.975$_{10}$-1	2.201$_{10}$ 1	3.138$_{10}$-1
35	2.500$_{10}$ 1	2.741$_{10}$-1	2.400$_{10}$ 1	2.883$_{10}$-1	2.301$_{10}$ 1	3.036$_{10}$-1
36	2.600$_{10}$ 1	2.664$_{10}$-1	2.500$_{10}$ 1	2.798$_{10}$-1	2.401$_{10}$ 1	2.941$_{10}$-1
37	2.700$_{10}$ 1	2.592$_{10}$-1	2.600$_{10}$ 1	2.718$_{10}$-1	2.500$_{10}$ 1	2.853$_{10}$-1
38	2.800$_{10}$ 1	2.525$_{10}$-1	2.700$_{10}$ 1	2.644$_{10}$-1	2.600$_{10}$ 1	2.771$_{10}$-1
39	2.900$_{10}$ 1	2.462$_{10}$-1	2.800$_{10}$ 1	2.575$_{10}$-1	2.700$_{10}$ 1	2.695$_{10}$-1
40	3.000$_{10}$ 1	2.404$_{10}$-1	2.900$_{10}$ 1	2.510$_{10}$-1	2.800$_{10}$ 1	2.624$_{10}$-1
41	3.100$_{10}$ 1	2.348$_{10}$-1	3.000$_{10}$ 1	2.449$_{10}$-1	2.900$_{10}$ 1	2.557$_{10}$-1
42	3.200$_{10}$ 1	2.296$_{10}$-1	3.100$_{10}$ 1	2.392$_{10}$-1	3.000$_{10}$ 1	2.494$_{10}$-1
43	3.300$_{10}$ 1	2.247$_{10}$-1	3.200$_{10}$ 1	2.338$_{10}$-1	3.100$_{10}$ 1	2.435$_{10}$-1
44	3.400$_{10}$ 1	2.201$_{10}$-1	3.300$_{10}$ 1	2.288$_{10}$-1	3.200$_{10}$ 1	2.380$_{10}$-1
45	3.500$_{10}$ 1	2.157$_{10}$-1	3.400$_{10}$ 1	2.240$_{10}$-1	3.300$_{10}$ 1	2.328$_{10}$-1
46	3.600$_{10}$ 1	2.115$_{10}$-1	3.500$_{10}$ 1	2.195$_{10}$-1	3.400$_{10}$ 1	2.278$_{10}$-1
47	3.700$_{10}$ 1	2.076$_{10}$-1	3.600$_{10}$ 1	2.152$_{10}$-1	3.500$_{10}$ 1	2.231$_{10}$-1
48	3.800$_{10}$ 1	2.038$_{10}$-1	3.700$_{10}$ 1	2.111$_{10}$-1	3.600$_{10}$ 1	2.187$_{10}$-1
49	3.900$_{10}$ 1	2.003$_{10}$-1	3.800$_{10}$ 1	2.072$_{10}$-1	3.700$_{10}$ 1	2.145$_{10}$-1
50	4.000$_{10}$ 1	1.969$_{10}$-1	3.900$_{10}$ 1	2.035$_{10}$-1	3.800$_{10}$ 1	2.105$_{10}$-1
51	4.100$_{10}$ 1	1.936$_{10}$-1	4.000$_{10}$ 1	2.000$_{10}$-1	3.900$_{10}$ 1	2.067$_{10}$-1

	n2 = 13		n2 = 14		n2 = 15	
n1	E[a]	D[a]/E[a]	E[a]	D[a]/E[a]	E[a]	D[a]/E[a]
1	1.125	9.876$_{10}$-1	1.118	9.889$_{10}$-1	1.111	9.899$_{10}$-1
2	1.263	9.747$_{10}$-1	1.247	9.773$_{10}$-1	1.233	9.795$_{10}$-1
3	1.415	9.612$_{10}$-1	1.389	9.651$_{10}$-1	1.366	9.685$_{10}$-1
4	1.583	9.469$_{10}$-1	1.545	9.523$_{10}$-1	1.511	9.570$_{10}$-1
5	1.769	9.319$_{10}$-1	1.717	9.388$_{10}$-1	1.671	9.447$_{10}$-1
6	1.975	9.158$_{10}$-1	1.907	9.243$_{10}$-1	1.847	9.317$_{10}$-1
7	2.204	8.988$_{10}$-1	2.117	9.090$_{10}$-1	2.041	9.178$_{10}$-1
8	2.457	8.807$_{10}$-1	2.348	8.926$_{10}$-1	2.254	9.030$_{10}$-1
9	2.737	8.615$_{10}$-1	2.604	8.752$_{10}$-1	2.489	8.872$_{10}$-1
10	3.047	8.412$_{10}$-1	2.887	8.568$_{10}$-1	2.747	8.705$_{10}$-1
11	3.390	8.198$_{10}$-1	3.199	8.373$_{10}$-1	3.032	8.527$_{10}$-1
12	3.769	7.974$_{10}$-1	3.542	8.168$_{10}$-1	3.345	8.339$_{10}$-1
13	4.184	7.740$_{10}$-1	3.920	7.952$_{10}$-1	3.689	8.141$_{10}$-1
14	4.640	7.498$_{10}$-1	4.334	7.728$_{10}$-1	4.066	7.933$_{10}$-1
15	5.138	7.249$_{10}$-1	4.787	7.495$_{10}$-1	4.478	7.717$_{10}$-1
16	5.679	6.995$_{10}$-1	5.280	7.256$_{10}$-1	4.929	7.493$_{10}$-1
17	6.264	6.737$_{10}$-1	5.816	7.011$_{10}$-1	5.418	7.262$_{10}$-1
18	6.894	6.478$_{10}$-1	6.394	6.762$_{10}$-1	5.948	7.025$_{10}$-1
19	7.567	6.220$_{10}$-1	7.015	6.512$_{10}$-1	6.520	6.785$_{10}$-1
20	8.281	5.963$_{10}$-1	7.679	6.261$_{10}$-1	7.134	6.542$_{10}$-1
21	9.036	5.712$_{10}$-1	8.383	6.012$_{10}$-1	7.789	6.299$_{10}$-1
22	9.828	5.466$_{10}$-1	9.128	5.766$_{10}$-1	8.484	6.056$_{10}$-1
23	1.065$_{10}$ 1	5.228$_{10}$-1	9.909	5.526$_{10}$-1	9.219	5.817$_{10}$-1
24	1.151$_{10}$ 1	4.999$_{10}$-1	1.073$_{10}$ 1	5.293$_{10}$-1	9.991	5.582$_{10}$-1
25	1.239$_{10}$ 1	4.780$_{10}$-1	1.157$_{10}$ 1	5.067$_{10}$-1	1.080$_{10}$ 1	5.353$_{10}$-1
26	1.330$_{10}$ 1	4.573$_{10}$-1	1.245$_{10}$ 1	4.851$_{10}$-1	1.163$_{10}$ 1	5.131$_{10}$-1
27	1.423$_{10}$ 1	4.376$_{10}$-1	1.334$_{10}$ 1	4.644$_{10}$-1	1.250$_{10}$ 1	4.917$_{10}$-1
28	1.517$_{10}$ 1	4.191$_{10}$-1	1.426$_{10}$ 1	4.449$_{10}$-1	1.339$_{10}$ 1	4.712$_{10}$-1
29	1.613$_{10}$ 1	4.018$_{10}$-1	1.520$_{10}$ 1	4.263$_{10}$-1	1.430$_{10}$ 1	4.517$_{10}$-1
30	1.709$_{10}$ 1	3.856$_{10}$-1	1.615$_{10}$ 1	4.089$_{10}$-1	1.523$_{10}$ 1	4.332$_{10}$-1
31	1.807$_{10}$ 1	3.705$_{10}$-1	1.711$_{10}$ 1	3.926$_{10}$-1	1.617$_{10}$ 1	4.158$_{10}$-1
32	1.905$_{10}$ 1	3.564$_{10}$-1	1.808$_{10}$ 1	3.773$_{10}$-1	1.713$_{10}$ 1	3.993$_{10}$-1
33	2.003$_{10}$ 1	3.434$_{10}$-1	1.906$_{10}$ 1	3.631$_{10}$-1	1.810$_{10}$ 1	3.839$_{10}$-1
34	2.102$_{10}$ 1	3.312$_{10}$-1	2.004$_{10}$ 1	3.498$_{10}$-1	1.907$_{10}$ 1	3.695$_{10}$-1
35	2.202$_{10}$ 1	3.200$_{10}$-1	2.103$_{10}$ 1	3.375$_{10}$-1	2.005$_{10}$ 1	3.561$_{10}$-1
36	2.301$_{10}$ 1	3.095$_{10}$-1	2.202$_{10}$ 1	3.260$_{10}$-1	2.104$_{10}$ 1	3.435$_{10}$-1
37	2.401$_{10}$ 1	2.998$_{10}$-1	2.301$_{10}$ 1	3.153$_{10}$-1	2.203$_{10}$ 1	3.318$_{10}$-1
38	2.501$_{10}$ 1	2.908$_{10}$-1	2.401$_{10}$ 1	3.053$_{10}$-1	2.302$_{10}$ 1	3.209$_{10}$-1
39	2.600$_{10}$ 1	2.823$_{10}$-1	2.501$_{10}$ 1	2.961$_{10}$-1	2.401$_{10}$ 1	3.107$_{10}$-1
40	2.700$_{10}$ 1	2.745$_{10}$-1	2.600$_{10}$ 1	2.874$_{10}$-1	2.501$_{10}$ 1	3.013$_{10}$-1
41	2.800$_{10}$ 1	2.671$_{10}$-1	2.700$_{10}$ 1	2.794$_{10}$-1	2.601$_{10}$ 1	2.924$_{10}$-1
42	2.900$_{10}$ 1	2.603$_{10}$-1	2.800$_{10}$ 1	2.718$_{10}$-1	2.700$_{10}$ 1	2.842$_{10}$-1
43	3.000$_{10}$ 1	2.538$_{10}$-1	2.900$_{10}$ 1	2.648$_{10}$-1	2.800$_{10}$ 1	2.764$_{10}$-1
44	3.100$_{10}$ 1	2.477$_{10}$-1	3.000$_{10}$ 1	2.581$_{10}$-1	2.900$_{10}$ 1	2.692$_{10}$-1
45	3.200$_{10}$ 1	2.420$_{10}$-1	3.100$_{10}$ 1	2.519$_{10}$-1	3.000$_{10}$ 1	2.624$_{10}$-1
46	3.300$_{10}$ 1	2.367$_{10}$-1	3.200$_{10}$ 1	2.460$_{10}$-1	3.100$_{10}$ 1	2.560$_{10}$-1
47	3.400$_{10}$ 1	2.316$_{10}$-1	3.300$_{10}$ 1	2.405$_{10}$-1	3.200$_{10}$ 1	2.500$_{10}$-1
48	3.500$_{10}$ 1	2.268$_{10}$-1	3.400$_{10}$ 1	2.353$_{10}$-1	3.300$_{10}$ 1	2.443$_{10}$-1
49	3.600$_{10}$ 1	2.222$_{10}$-1	3.500$_{10}$ 1	2.303$_{10}$-1	3.400$_{10}$ 1	2.389$_{10}$-1
50	3.700$_{10}$ 1	2.179$_{10}$-1	3.600$_{10}$ 1	2.257$_{10}$-1	3.500$_{10}$ 1	2.339$_{10}$-1
51	3.800$_{10}$ 1	2.138$_{10}$-1	3.700$_{10}$ 1	2.212$_{10}$-1	3.600$_{10}$ 1	2.291$_{10}$-1

	n2 = 16		n2 = 17		n2 = 18	
n1	E[a]	D[a]/E[a]	E[a]	D[a]/E[a]	E[a]	D[a]/E[a]
1	1.105	9.909$_{10}$-1	1.100	9.917$_{10}$-1	1.095	9.924$_{10}$-1
2	1.220	9.814$_{10}$-1	1.209	9.831$_{10}$-1	1.198	9.845$_{10}$-1
3	1.345	9.715$_{10}$-1	1.327	9.740$_{10}$-1	1.310	9.762$_{10}$-1
4	1.482	9.610$_{10}$-1	1.455	9.645$_{10}$-1	1.431	9.675$_{10}$-1
5	1.631	9.499$_{10}$-1	1.595	9.543$_{10}$-1	1.563	9.582$_{10}$-1
6	1.795	9.380$_{10}$-1	1.748	9.436$_{10}$-1	1.706	9.484$_{10}$-1
7	1.974	9.254$_{10}$-1	1.915	9.321$_{10}$-1	1.862	9.380$_{10}$-1
8	2.171	9.120$_{10}$-1	2.098	9.199$_{10}$-1	2.032	9.268$_{10}$-1
9	2.387	8.977$_{10}$-1	2.298	9.068$_{10}$-1	2.218	9.149$_{10}$-1
10	2.625	8.824$_{10}$-1	2.517	8.929$_{10}$-1	2.421	9.022$_{10}$-1
11	2.885	8.662$_{10}$-1	2.757	8.781$_{10}$-1	2.643	8.887$_{10}$-1
12	3.172	8.490$_{10}$-1	3.020	8.624$_{10}$-1	2.885	8.742$_{10}$-1
13	3.486	8.308$_{10}$-1	3.308	8.457$_{10}$-1	3.150	8.589$_{10}$-1
14	3.830	8.117$_{10}$-1	3.623	8.281$_{10}$-1	3.440	8.427$_{10}$-1
15	4.207	7.917$_{10}$-1	3.968	8.096$_{10}$-1	3.757	8.256$_{10}$-1
16	4.618	7.708$_{10}$-1	4.344	7.901$_{10}$-1	4.102	8.076$_{10}$-1
17	5.066	7.491$_{10}$-1	4.754	7.699$_{10}$-1	4.478	7.888$_{10}$-1
18	5.552	7.267$_{10}$-1	5.200	7.489$_{10}$-1	4.886	7.692$_{10}$-1
19	6.077	7.039$_{10}$-1	5.682	7.273$_{10}$-1	5.330	7.488$_{10}$-1
20	6.643	6.806$_{10}$-1	6.203	7.051$_{10}$-1	5.809	7.278$_{10}$-1
21	7.250	6.570$_{10}$-1	6.764	6.825$_{10}$-1	6.326	7.062$_{10}$-1
22	7.897	6.334$_{10}$-1	7.364	6.596$_{10}$-1	6.881	6.843$_{10}$-1
23	8.584	6.098$_{10}$-1	8.004	6.366$_{10}$-1	7.476	6.620$_{10}$-1
24	9.310	5.864$_{10}$-1	8.683	6.136$_{10}$-1	8.109	6.396$_{10}$-1
25	1.007$_{10}$ 1	5.634$_{10}$-1	9.400	5.907$_{10}$-1	8.781	6.171$_{10}$-1
26	1.087$_{10}$ 1	5.409$_{10}$-1	1.015$_{10}$ 1	5.682$_{10}$-1	9.490	5.948$_{10}$-1
27	1.170$_{10}$ 1	5.190$_{10}$-1	1.094$_{10}$ 1	5.461$_{10}$-1	1.024$_{10}$ 1	5.727$_{10}$-1
28	1.255$_{10}$ 1	4.979$_{10}$-1	1.176$_{10}$ 1	5.245$_{10}$-1	1.101$_{10}$ 1	5.510$_{10}$-1
29	1.343$_{10}$ 1	4.776$_{10}$-1	1.261$_{10}$ 1	5.037$_{10}$-1	1.183$_{10}$ 1	5.298$_{10}$-1
30	1.434$_{10}$ 1	4.582$_{10}$-1	1.348$_{10}$ 1	4.836$_{10}$-1	1.266$_{10}$ 1	5.092$_{10}$-1
31	1.526$_{10}$ 1	4.397$_{10}$-1	1.438$_{10}$ 1	4.643$_{10}$-1	1.353$_{10}$ 1	4.893$_{10}$-1
32	1.620$_{10}$ 1	4.223$_{10}$-1	1.529$_{10}$ 1	4.459$_{10}$-1	1.442$_{10}$ 1	4.701$_{10}$-1
33	1.715$_{10}$ 1	4.058$_{10}$-1	1.623$_{10}$ 1	4.285$_{10}$-1	1.533$_{10}$ 1	4.519$_{10}$-1
34	1.811$_{10}$ 1	3.903$_{10}$-1	1.717$_{10}$ 1	4.120$_{10}$-1	1.625$_{10}$ 1	4.344$_{10}$-1
35	1.908$_{10}$ 1	3.757$_{10}$-1	1.813$_{10}$ 1	3.964$_{10}$-1	1.720$_{10}$ 1	4.179$_{10}$-1
36	2.006$_{10}$ 1	3.621$_{10}$-1	1.910$_{10}$ 1	3.817$_{10}$-1	1.815$_{10}$ 1	4.022$_{10}$-1
37	2.104$_{10}$ 1	3.494$_{10}$-1	2.007$_{10}$ 1	3.680$_{10}$-1	1.911$_{10}$ 1	3.875$_{10}$-1
38	2.203$_{10}$ 1	3.375$_{10}$-1	2.105$_{10}$ 1	3.551$_{10}$-1	2.008$_{10}$ 1	3.736$_{10}$-1
39	2.302$_{10}$ 1	3.264$_{10}$-1	2.204$_{10}$ 1	3.430$_{10}$-1	2.106$_{10}$ 1	3.606$_{10}$-1
40	2.402$_{10}$ 1	3.160$_{10}$-1	2.303$_{10}$ 1	3.317$_{10}$-1	2.205$_{10}$ 1	3.484$_{10}$-1
41	2.501$_{10}$ 1	3.063$_{10}$-1	2.402$_{10}$ 1	3.212$_{10}$-1	2.303$_{10}$ 1	3.369$_{10}$-1
42	2.601$_{10}$ 1	2.973$_{10}$-1	2.501$_{10}$ 1	3.113$_{10}$-1	2.402$_{10}$ 1	3.262$_{10}$-1
43	2.701$_{10}$ 1	2.888$_{10}$-1	2.601$_{10}$ 1	3.021$_{10}$-1	2.502$_{10}$ 1	3.162$_{10}$-1
44	2.800$_{10}$ 1	2.809$_{10}$-1	2.701$_{10}$ 1	2.934$_{10}$-1	2.601$_{10}$ 1	3.068$_{10}$-1
45	2.900$_{10}$ 1	2.735$_{10}$-1	2.801$_{10}$ 1	2.853$_{10}$-1	2.701$_{10}$ 1	2.979$_{10}$-1
46	3.000$_{10}$ 1	2.665$_{10}$-1	2.900$_{10}$ 1	2.778$_{10}$-1	2.801$_{10}$ 1	2.897$_{10}$-1
47	3.100$_{10}$ 1	2.600$_{10}$-1	3.000$_{10}$ 1	2.706$_{10}$-1	2.900$_{10}$ 1	2.819$_{10}$-1
48	3.200$_{10}$ 1	2.538$_{10}$-1	3.100$_{10}$ 1	2.639$_{10}$-1	3.000$_{10}$ 1	2.747$_{10}$-1
49	3.300$_{10}$ 1	2.480$_{10}$-1	3.200$_{10}$ 1	2.576$_{10}$-1	3.100$_{10}$ 1	2.678$_{10}$-1
50	3.400$_{10}$ 1	2.425$_{10}$-1	3.300$_{10}$ 1	2.517$_{10}$-1	3.200$_{10}$ 1	2.614$_{10}$-1
51	3.500$_{10}$ 1	2.373$_{10}$-1	3.[illegible]00$_{10}$ 1	2.460$_{10}$-1	3.300$_{10}$ 1	2.553$_{10}$-1

	n2 = 19		n2 = 20		n2 = 21	
n1	E[a]	D[a]/E[a]	E[a]	D[a]/E[a]	E[a]	D[a]/E[a]
1	1.091	$9.930_{10}-1$	1.087	$9.936_{10}-1$	1.083	$9.941_{10}-1$
2	1.189	$9.858_{10}-1$	1.181	$9.869_{10}-1$	1.173	$9.879_{10}-1$
3	1.295	$9.782_{10}-1$	1.281	$9.799_{10}-1$	1.269	$9.814_{10}-1$
4	1.410	$9.702_{10}-1$	1.390	$9.725_{10}-1$	1.372	$9.746_{10}-1$
5	1.534	$9.617_{10}-1$	1.507	$9.647_{10}-1$	1.484	$9.674_{10}-1$
6	1.668	$9.527_{10}-1$	1.635	$9.565_{10}-1$	1.604	$9.598_{10}-1$
7	1.815	$9.431_{10}-1$	1.772	$9.477_{10}-1$	1.734	$9.517_{10}-1$
8	1.974	$9.329_{10}-1$	1.921	$9.383_{10}-1$	1.874	$9.431_{10}-1$
9	2.147	$9.220_{10}-1$	2.083	$9.283_{10}-1$	2.026	$9.339_{10}-1$
10	2.336	$9.104_{10}-1$	2.260	$9.177_{10}-1$	2.191	$9.241_{10}-1$
11	2.542	$8.980_{10}-1$	2.451	$9.063_{10}-1$	2.370	$9.137_{10}-1$
12	2.766	$8.848_{10}-1$	2.659	$8.941_{10}-1$	2.564	$9.025_{10}-1$
13	3.011	$8.707_{10}-1$	2.886	$8.812_{10}-1$	2.775	$8.906_{10}-1$
14	3.278	$8.558_{10}-1$	3.133	$8.675_{10}-1$	3.004	$8.780_{10}-1$
15	3.569	$8.400_{10}-1$	3.402	$8.529_{10}-1$	3.252	$8.645_{10}-1$
16	3.886	$8.234_{10}-1$	3.694	$8.376_{10}-1$	3.523	$8.503_{10}-1$
17	4.232	$8.059_{10}-1$	4.013	$8.213_{10}-1$	3.817	$8.353_{10}-1$
18	4.607	$7.876_{10}-1$	4.358	$8.043_{10}-1$	4.136	$8.194_{10}-1$
19	5.015	$7.685_{10}-1$	4.734	$7.864_{10}-1$	4.482	$8.028_{10}-1$
20	5.456	$7.487_{10}-1$	5.140	$7.679_{10}-1$	4.857	$7.854_{10}-1$
21	5.933	$7.283_{10}-1$	5.580	$7.486_{10}-1$	5.263	$7.673_{10}-1$
22	6.446	$7.073_{10}-1$	6.054	$7.287_{10}-1$	5.700	$7.485_{10}-1$
23	6.997	$6.859_{10}-1$	6.563	$7.083_{10}-1$	6.172	$7.291_{10}-1$
24	7.586	$6.642_{10}-1$	7.110	$6.875_{10}-1$	6.678	$7.092_{10}-1$
25	8.213	$6.423_{10}-1$	7.694	$6.663_{10}-1$	7.221	$6.889_{10}-1$
26	8.877	$6.204_{10}-1$	8.315	$6.449_{10}-1$	7.800	$6.682_{10}-1$
27	9.579	$5.985_{10}-1$	8.973	$6.235_{10}-1$	8.415	$6.474_{10}-1$
28	$1.032_{10}\,1$	$5.769_{10}-1$	9.668	$6.021_{10}-1$	9.067	$6.264_{10}-1$
29	$1.109_{10}\,1$	$5.556_{10}-1$	$1.040_{10}\,1$	$5.808_{10}-1$	9.755	$6.054_{10}-1$
30	$1.189_{10}\,1$	$5.347_{10}-1$	$1.116_{10}\,1$	$5.599_{10}-1$	$1.048_{10}\,1$	$5.846_{10}-1$
31	$1.272_{10}\,1$	$5.144_{10}-1$	$1.196_{10}\,1$	$5.393_{10}-1$	$1.123_{10}\,1$	$5.640_{10}-1$
32	$1.358_{10}\,1$	$4.947_{10}-1$	$1.278_{10}\,1$	$5.193_{10}-1$	$1.202_{10}\,1$	$5.437_{10}-1$
33	$1.446_{10}\,1$	$4.757_{10}-1$	$1.363_{10}\,1$	$4.998_{10}-1$	$1.284_{10}\,1$	$5.239_{10}-1$
34	$1.536_{10}\,1$	$4.575_{10}-1$	$1.451_{10}\,1$	$4.810_{10}-1$	$1.368_{10}\,1$	$5.046_{10}-1$
35	$1.628_{10}\,1$	$4.401_{10}-1$	$1.540_{10}\,1$	$4.629_{10}-1$	$1.455_{10}\,1$	$4.860_{10}-1$
36	$1.722_{10}\,1$	$4.236_{10}-1$	$1.632_{10}\,1$	$4.455_{10}-1$	$1.544_{10}\,1$	$4.680_{10}-1$
37	$1.817_{10}\,1$	$4.079_{10}-1$	$1.725_{10}\,1$	$4.290_{10}-1$	$1.635_{10}\,1$	$4.508_{10}-1$
38	$1.913_{10}\,1$	$3.931_{10}-1$	$1.819_{10}\,1$	$4.133_{10}-1$	$1.727_{10}\,1$	$4.343_{10}-1$
39	$2.010_{10}\,1$	$3.791_{10}-1$	$1.915_{10}\,1$	$3.984_{10}-1$	$1.821_{10}\,1$	$4.185_{10}-1$
40	$2.107_{10}\,1$	$3.659_{10}-1$	$2.011_{10}\,1$	$3.844_{10}-1$	$1.916_{10}\,1$	$4.036_{10}-1$
41	$2.205_{10}\,1$	$3.536_{10}-1$	$2.108_{10}\,1$	$3.711_{10}-1$	$2.013_{10}\,1$	$3.895_{10}-1$
42	$2.304_{10}\,1$	$3.420_{10}-1$	$2.206_{10}\,1$	$3.586_{10}-1$	$2.110_{10}\,1$	$3.762_{10}-1$
43	$2.403_{10}\,1$	$3.311_{10}-1$	$2.305_{10}\,1$	$3.469_{10}-1$	$2.207_{10}\,1$	$3.636_{10}-1$
44	$2.502_{10}\,1$	$3.209_{10}-1$	$2.403_{10}\,1$	$3.359_{10}-1$	$2.305_{10}\,1$	$3.517_{10}-1$
45	$2.602_{10}\,1$	$3.113_{10}-1$	$2.503_{10}\,1$	$3.255_{10}-1$	$2.404_{10}\,1$	$3.405_{10}-1$
46	$2.701_{10}\,1$	$3.024_{10}-1$	$2.602_{10}\,1$	$3.158_{10}-1$	$2.503_{10}\,1$	$3.301_{10}-1$
47	$2.801_{10}\,1$	$2.939_{10}-1$	$2.701_{10}\,1$	$3.067_{10}-1$	$2.602_{10}\,1$	$3.202_{10}-1$
48	$2.901_{10}\,1$	$2.860_{10}-1$	$2.801_{10}\,1$	$2.981_{10}-1$	$2.702_{10}\,1$	$3.109_{10}-1$
49	$3.000_{10}\,1$	$2.786_{10}-1$	$2.901_{10}\,1$	$2.901_{10}-1$	$2.801_{10}\,1$	$3.022_{10}-1$
50	$3.100_{10}\,1$	$2.716_{10}-1$	$3.000_{10}\,1$	$2.825_{10}-1$	$2.901_{10}\,1$	$2.940_{10}-1$
51	$3.200_{10}\,1$	$2.650_{10}-1$	$3.100_{10}\,1$	$2.754_{10}-1$	$3.001_{10}\,1$	$2.863_{10}-1$

	n2 = 22		n2 = 23		n2 = 24	
n1	E[a]	D[a]/E[a]	E[a]	D[a]/E[a]	E[a]	D[a]/E[a]
1	1.080	9.945$_{10}$-1	1.077	9.949$_{10}$-1	1.074	9.952$_{10}$-1
2	1.166	9.888$_{10}$-1	1.159	9.896$_{10}$-1	1.153	9.903$_{10}$-1
3	1.257	9.828$_{10}$-1	1.247	9.840$_{10}$-1	1.237	9.851$_{10}$-1
4	1.356	9.765$_{10}$-1	1.341	9.782$_{10}$-1	1.327	9.797$_{10}$-1
5	1.462	9.698$_{10}$-1	1.442	9.720$_{10}$-1	1.424	9.739$_{10}$-1
6	1.576	9.628$_{10}$-1	1.550	9.655$_{10}$-1	1.527	9.679$_{10}$-1
7	1.699	9.553$_{10}$-1	1.667	9.586$_{10}$-1	1.638	9.615$_{10}$-1
8	1.831	9.474$_{10}$-1	1.792	9.512$_{10}$-1	1.756	9.547$_{10}$-1
9	1.974	9.389$_{10}$-1	1.927	9.434$_{10}$-1	1.884	9.474$_{10}$-1
10	2.129	9.299$_{10}$-1	2.073	9.351$_{10}$-1	2.022	9.397$_{10}$-1
11	2.296	9.203$_{10}$-1	2.230	9.262$_{10}$-1	2.170	9.315$_{10}$-1
12	2.478	9.100$_{10}$-1	2.400	9.167$_{10}$-1	2.329	9.227$_{10}$-1
13	2.674	8.990$_{10}$-1	2.584	9.066$_{10}$-1	2.502	9.134$_{10}$-1
14	2.887	8.874$_{10}$-1	2.783	8.958$_{10}$-1	2.688	9.034$_{10}$-1
15	3.119	8.750$_{10}$-1	2.998	8.844$_{10}$-1	2.889	8.928$_{10}$-1
16	3.369	8.618$_{10}$-1	3.231	8.722$_{10}$-1	3.106	8.816$_{10}$-1
17	3.641	8.479$_{10}$-1	3.484	8.593$_{10}$-1	3.342	8.696$_{10}$-1
18	3.937	8.332$_{10}$-1	3.757	8.456$_{10}$-1	3.596	8.569$_{10}$-1
19	4.256	8.177$_{10}$-1	4.054	8.312$_{10}$-1	3.871	8.436$_{10}$-1
20	4.603	8.015$_{10}$-1	4.374	8.161$_{10}$-1	4.168	8.294$_{10}$-1
21	4.978	7.845$_{10}$-1	4.721	8.002$_{10}$-1	4.490	8.146$_{10}$-1
22	5.382	7.668$_{10}$-1	5.095	7.836$_{10}$-1	4.837	7.990$_{10}$-1
23	5.818	7.485$_{10}$-1	5.499	7.663$_{10}$-1	5.211	7.828$_{10}$-1
24	6.288	7.295$_{10}$-1	5.934	7.484$_{10}$-1	5.614	7.659$_{10}$-1
25	6.791	7.101$_{10}$-1	6.401	7.299$_{10}$-1	6.047	7.484$_{10}$-1
26	7.330	6.902$_{10}$-1	6.902	7.109$_{10}$-1	6.512	7.303$_{10}$-1
27	7.904	6.701$_{10}$-1	7.436	6.915$_{10}$-1	7.010	7.117$_{10}$-1
28	8.514	6.496$_{10}$-1	8.007	6.718$_{10}$-1	7.542	6.927$_{10}$-1
29	9.160	6.291$_{10}$-1	8.612	6.518$_{10}$-1	8.108	6.734$_{10}$-1
30	9.842	6.085$_{10}$-1	9.253	6.316$_{10}$-1	8.708	6.538$_{10}$-1
31	1.056$_{10}$ 1	5.881$_{10}$-1	9.928	6.115$_{10}$-1	9.344	6.341$_{10}$-1
32	1.131$_{10}$ 1	5.678$_{10}$-1	1.064$_{10}$ 1	5.914$_{10}$-1	1.001$_{10}$ 1	6.143$_{10}$-1
33	1.209$_{10}$ 1	5.479$_{10}$-1	1.138$_{10}$ 1	5.714$_{10}$-1	1.072$_{10}$ 1	5.945$_{10}$-1
34	1.290$_{10}$ 1	5.283$_{10}$-1	1.215$_{10}$ 1	5.518$_{10}$-1	1.145$_{10}$ 1	5.749$_{10}$-1
35	1.373$_{10}$ 1	5.093$_{10}$-1	1.296$_{10}$ 1	5.325$_{10}$-1	1.222$_{10}$ 1	5.555$_{10}$-1
36	1.460$_{10}$ 1	4.908$_{10}$-1	1.379$_{10}$ 1	5.137$_{10}$-1	1.302$_{10}$ 1	5.365$_{10}$-1
37	1.548$_{10}$ 1	4.729$_{10}$-1	1.464$_{10}$ 1	4.953$_{10}$-1	1.384$_{10}$ 1	5.179$_{10}$-1
38	1.638$_{10}$ 1	4.557$_{10}$-1	1.552$_{10}$ 1	4.776$_{10}$-1	1.469$_{10}$ 1	4.997$_{10}$-1
39	1.730$_{10}$ 1	4.393$_{10}$-1	1.642$_{10}$ 1	4.605$_{10}$-1	1.556$_{10}$ 1	4.821$_{10}$-1
40	1.824$_{10}$ 1	4.236$_{10}$-1	1.733$_{10}$ 1	4.441$_{10}$-1	1.645$_{10}$ 1	4.651$_{10}$-1
41	1.918$_{10}$ 1	4.086$_{10}$-1	1.826$_{10}$ 1	4.285$_{10}$-1	1.736$_{10}$ 1	4.488$_{10}$-1
42	2.014$_{10}$ 1	3.945$_{10}$-1	1.921$_{10}$ 1	4.135$_{10}$-1	1.829$_{10}$ 1	4.331$_{10}$-1
43	2.111$_{10}$ 1	3.810$_{10}$-1	2.016$_{10}$ 1	3.993$_{10}$-1	1.923$_{10}$ 1	4.182$_{10}$-1
44	2.208$_{10}$ 1	3.684$_{10}$-1	2.112$_{10}$ 1	3.858$_{10}$-1	2.018$_{10}$ 1	4.039$_{10}$-1
45	2.306$_{10}$ 1	3.564$_{10}$-1	2.209$_{10}$ 1	3.730$_{10}$-1	2.114$_{10}$ 1	3.904$_{10}$-1
46	2.405$_{10}$ 1	3.451$_{10}$-1	2.307$_{10}$ 1	3.609$_{10}$-1	2.211$_{10}$ 1	3.776$_{10}$-1
47	2.503$_{10}$ 1	3.345$_{10}$-1	2.405$_{10}$ 1	3.496$_{10}$-1	2.308$_{10}$ 1	3.654$_{10}$-1
48	2.603$_{10}$ 1	3.245$_{10}$-1	2.504$_{10}$ 1	3.388$_{10}$-1	2.406$_{10}$ 1	3.539$_{10}$-1
49	2.702$_{10}$ 1	3.151$_{10}$-1	2.603$_{10}$ 1	3.287$_{10}$-1	2.505$_{10}$ 1	3.431$_{10}$-1
50	2.801$_{10}$ 1	3.063$_{10}$-1	2.702$_{10}$ 1	3.192$_{10}$-1	2.603$_{10}$ 1	3.328$_{10}$-1
51	2.901$_{10}$ 1	2.979$_{10}$-1	2.802$_{10}$ 1	3.102$_{10}$-1	2.703$_{10}$ 1	3.232$_{10}$-1

	n2 = 25		n2 = 26		n2 = 27	
n1	E[a]	D[a]/E[a]	E[a]	D[a]/E[a]	E[a]	D[a]/E[a]
1	1.071	$9.955_{10}-1$	1.069	$9.958_{10}-1$	1.067	$9.961_{10}-1$
2	1.147	$9.909_{10}-1$	1.142	$9.915_{10}-1$	1.137	$9.920_{10}-1$
3	1.228	$9.861_{10}-1$	1.220	$9.870_{10}-1$	1.212	$9.878_{10}-1$
4	1.315	$9.810_{10}-1$	1.303	$9.822_{10}-1$	1.292	$9.833_{10}-1$
5	1.407	$9.757_{10}-1$	1.391	$9.773_{10}-1$	1.377	$9.787_{10}-1$
6	1.505	$9.700_{10}-1$	1.485	$9.720_{10}-1$	1.467	$9.738_{10}-1$
7	1.611	$9.641_{10}-1$	1.586	$9.664_{10}-1$	1.563	$9.686_{10}-1$
8	1.724	$9.578_{10}-1$	1.694	$9.606_{10}-1$	1.666	$9.631_{10}-1$
9	1.845	$9.510_{10}-1$	1.809	$9.543_{10}-1$	1.775	$9.573_{10}-1$
10	1.975	$9.439_{10}-1$	1.932	$9.477_{10}-1$	1.893	$9.511_{10}-1$
11	2.115	$9.363_{10}-1$	2.064	$9.406_{10}-1$	2.018	$9.445_{10}-1$
12	2.265	$9.281_{10}-1$	2.207	$9.331_{10}-1$	2.153	$9.375_{10}-1$
13	2.427	$9.195_{10}-1$	2.359	$9.250_{10}-1$	2.297	$9.300_{10}-1$
14	2.602	$9.103_{10}-1$	2.524	$9.165_{10}-1$	2.452	$9.221_{10}-1$
15	2.790	$9.005_{10}-1$	2.700	$9.074_{10}-1$	2.618	$9.137_{10}-1$
16	2.993	$8.900_{10}-1$	2.891	$8.977_{10}-1$	2.797	$9.047_{10}-1$
17	3.213	$8.790_{10}-1$	3.096	$8.874_{10}-1$	2.990	$8.951_{10}-1$
18	3.450	$8.672_{10}-1$	3.317	$8.765_{10}-1$	3.197	$8.850_{10}-1$
19	3.706	$8.547_{10}-1$	3.556	$8.649_{10}-1$	3.420	$8.742_{10}-1$
20	3.982	$8.416_{10}-1$	3.814	$8.527_{10}-1$	3.661	$8.628_{10}-1$
21	4.281	$8.277_{10}-1$	4.091	$8.398_{10}-1$	3.920	$8.508_{10}-1$
22	4.603	$8.132_{10}-1$	4.391	$8.262_{10}-1$	4.198	$8.381_{10}-1$
23	4.950	$7.980_{10}-1$	4.713	$8.119_{10}-1$	4.499	$8.247_{10}-1$
24	5.324	$7.820_{10}-1$	5.061	$7.969_{10}-1$	4.822	$8.107_{10}-1$
25	5.726	$7.655_{10}-1$	5.435	$7.813_{10}-1$	5.170	$7.960_{10}-1$
26	6.158	$7.483_{10}-1$	5.836	$7.651_{10}-1$	5.543	$7.807_{10}-1$
27	6.621	$7.306_{10}-1$	6.267	$7.483_{10}-1$	5.944	$7.648_{10}-1$
28	7.116	$7.125_{10}-1$	6.728	$7.310_{10}-1$	6.374	$7.483_{10}-1$
29	7.645	$6.939_{10}-1$	7.221	$7.132_{10}-1$	6.833	$7.313_{10}-1$
30	8.207	$6.749_{10}-1$	7.746	$6.949_{10}-1$	7.324	$7.138_{10}-1$
31	8.803	$6.557_{10}-1$	8.305	$6.764_{10}-1$	7.846	$6.959_{10}-1$
32	9.434	$6.363_{10}-1$	8.897	$6.575_{10}-1$	8.402	$6.777_{10}-1$
33	$1.010_{10}\,1$	$6.169_{10}-1$	9.523	$6.385_{10}-1$	8.990	$6.592_{10}-1$
34	$1.080_{10}\,1$	$5.975_{10}-1$	$1.018_{10}\,1$	$6.194_{10}-1$	9.611	$6.406_{10}-1$
35	$1.153_{10}\,1$	$5.782_{10}-1$	$1.087_{10}\,1$	$6.003_{10}-1$	$1.027_{10}\,1$	$6.218_{10}-1$
36	$1.229_{10}\,1$	$5.591_{10}-1$	$1.160_{10}\,1$	$5.813_{10}-1$	$1.095_{10}\,1$	$6.030_{10}-1$
37	$1.308_{10}\,1$	$5.403_{10}-1$	$1.235_{10}\,1$	$5.625_{10}-1$	$1.167_{10}\,1$	$5.843_{10}-1$
38	$1.389_{10}\,1$	$5.219_{10}-1$	$1.314_{10}\,1$	$5.439_{10}-1$	$1.242_{10}\,1$	$5.657_{10}-1$
39	$1.474_{10}\,1$	$5.039_{10}-1$	$1.395_{10}\,1$	$5.257_{10}-1$	$1.320_{10}\,1$	$5.474_{10}-1$
40	$1.560_{10}\,1$	$4.865_{10}-1$	$1.478_{10}\,1$	$5.079_{10}-1$	$1.400_{10}\,1$	$5.294_{10}-1$
41	$1.649_{10}\,1$	$4.696_{10}-1$	$1.564_{10}\,1$	$4.906_{10}-1$	$1.483_{10}\,1$	$5.118_{10}-1$
42	$1.739_{10}\,1$	$4.533_{10}-1$	$1.653_{10}\,1$	$4.738_{10}-1$	$1.569_{10}\,1$	$4.946_{10}-1$
43	$1.831_{10}\,1$	$4.377_{10}-1$	$1.743_{10}\,1$	$4.576_{10}-1$	$1.656_{10}\,1$	$4.779_{10}-1$
44	$1.925_{10}\,1$	$4.227_{10}-1$	$1.834_{10}\,1$	$4.421_{10}-1$	$1.746_{10}\,1$	$4.618_{10}-1$
45	$2.020_{10}\,1$	$4.085_{10}-1$	$1.927_{10}\,1$	$4.271_{10}-1$	$1.837_{10}\,1$	$4.463_{10}-1$
46	$2.115_{10}\,1$	$3.949_{10}-1$	$2.022_{10}\,1$	$4.128_{10}-1$	$1.930_{10}\,1$	$4.314_{10}-1$
47	$2.212_{10}\,1$	$3.820_{10}-1$	$2.117_{10}\,1$	$3.992_{10}-1$	$2.024_{10}\,1$	$4.171_{10}-1$
48	$2.309_{10}\,1$	$3.697_{10}-1$	$2.213_{10}\,1$	$3.863_{10}-1$	$2.119_{10}\,1$	$4.034_{10}-1$
49	$2.407_{10}\,1$	$3.582_{10}-1$	$2.310_{10}\,1$	$3.740_{10}-1$	$2.215_{10}\,1$	$3.905_{10}-1$
50	$2.505_{10}\,1$	$3.472_{10}-1$	$2.408_{10}\,1$	$3.623_{10}-1$	$2.311_{10}\,1$	$3.781_{10}-1$
51	$2.604_{10}\,1$	$3.369_{10}-1$	$2.506_{10}\,1$	$3.513_{10}-1$	$2.409_{10}\,1$	$3.664_{10}-1$

	n2 = 28		n2 = 29		n2 = 30	
n1	E[a]	D[a]/E[a]	E[a]	D[a]/E[a]	E[a]	D[a]/E[a]
1	1.065	$9.963_{10}{-1}$	1.062	$9.965_{10}{-1}$	1.061	$9.967_{10}{-1}$
2	1.133	$9.925_{10}{-1}$	1.129	$9.929_{10}{-1}$	1.125	$9.933_{10}{-1}$
3	1.205	$9.885_{10}{-1}$	1.198	$9.892_{10}{-1}$	1.192	$9.898_{10}{-1}$
4	1.282	$9.844_{10}{-1}$	1.272	$9.853_{10}{-1}$	1.264	$9.861_{10}{-1}$
5	1.363	$9.800_{10}{-1}$	1.351	$9.812_{10}{-1}$	1.339	$9.823_{10}{-1}$
6	1.450	$9.754_{10}{-1}$	1.434	$9.769_{10}{-1}$	1.419	$9.782_{10}{-1}$
7	1.542	$9.705_{10}{-1}$	1.522	$9.723_{10}{-1}$	1.504	$9.739_{10}{-1}$
8	1.640	$9.654_{10}{-1}$	1.616	$9.675_{10}{-1}$	1.594	$9.694_{10}{-1}$
9	1.745	$9.600_{10}{-1}$	1.716	$9.624_{10}{-1}$	1.690	$9.646_{10}{-1}$
10	1.856	$9.542_{10}{-1}$	1.823	$9.570_{10}{-1}$	1.792	$9.596_{10}{-1}$
11	1.976	$9.481_{10}{-1}$	1.936	$9.513_{10}{-1}$	1.900	$9.542_{10}{-1}$
12	2.103	$9.415_{10}{-1}$	2.058	$9.452_{10}{-1}$	2.016	$9.486_{10}{-1}$
13	2.240	$9.346_{10}{-1}$	2.187	$9.387_{10}{-1}$	2.139	$9.425_{10}{-1}$
14	2.386	$9.272_{10}{-1}$	2.326	$9.319_{10}{-1}$	2.270	$9.361_{10}{-1}$
15	2.543	$9.194_{10}{-1}$	2.474	$9.245_{10}{-1}$	2.411	$9.293_{10}{-1}$
16	2.712	$9.110_{10}{-1}$	2.634	$9.168_{10}{-1}$	2.561	$9.220_{10}{-1}$
17	2.893	$9.021_{10}{-1}$	2.804	$9.085_{10}{-1}$	2.722	$9.143_{10}{-1}$
18	3.087	$8.927_{10}{-1}$	2.987	$8.997_{10}{-1}$	2.895	$9.061_{10}{-1}$
19	3.296	$8.827_{10}{-1}$	3.183	$8.904_{10}{-1}$	3.080	$8.974_{10}{-1}$
20	3.521	$8.720_{10}{-1}$	3.394	$8.805_{10}{-1}$	3.278	$8.882_{10}{-1}$
21	3.763	$8.608_{10}{-1}$	3.621	$8.700_{10}{-1}$	3.491	$8.784_{10}{-1}$
22	4.024	$8.489_{10}{-1}$	3.864	$8.589_{10}{-1}$	3.719	$8.681_{10}{-1}$
23	4.304	$8.364_{10}{-1}$	4.126	$8.472_{10}{-1}$	3.964	$8.571_{10}{-1}$
24	4.605	$8.233_{10}{-1}$	4.407	$8.349_{10}{-1}$	4.227	$8.456_{10}{-1}$
25	4.929	$8.095_{10}{-1}$	4.709	$8.220_{10}{-1}$	4.509	$8.335_{10}{-1}$
26	5.277	$7.951_{10}{-1}$	5.034	$8.084_{10}{-1}$	4.812	$8.207_{10}{-1}$
27	5.650	$7.801_{10}{-1}$	5.382	$7.943_{10}{-1}$	5.137	$8.074_{10}{-1}$
28	6.050	$7.645_{10}{-1}$	5.755	$7.795_{10}{-1}$	5.485	$7.935_{10}{-1}$
29	6.479	$7.483_{10}{-1}$	6.155	$7.642_{10}{-1}$	5.858	$7.790_{10}{-1}$
30	6.937	$7.316_{10}{-1}$	6.582	$7.483_{10}{-1}$	6.257	$7.639_{10}{-1}$
31	7.425	$7.145_{10}{-1}$	7.038	$7.319_{10}{-1}$	6.684	$7.483_{10}{-1}$
32	7.945	$6.969_{10}{-1}$	7.525	$7.151_{10}{-1}$	7.139	$7.322_{10}{-1}$
33	8.497	$6.790_{10}{-1}$	8.042	$6.978_{10}{-1}$	7.623	$7.156_{10}{-1}$
34	9.081	$6.609_{10}{-1}$	8.591	$6.803_{10}{-1}$	8.138	$6.987_{10}{-1}$
35	9.698	$6.425_{10}{-1}$	9.172	$6.624_{10}{-1}$	8.683	$6.814_{10}{-1}$
36	$1.035_{10}\,1$	$6.241_{10}{-1}$	9.785	$6.444_{10}{-1}$	9.261	$6.639_{10}{-1}$
37	$1.103_{10}\,1$	$6.056_{10}{-1}$	$1.043_{10}\,1$	$6.262_{10}{-1}$	9.870	$6.462_{10}{-1}$
38	$1.174_{10}\,1$	$5.871_{10}{-1}$	$1.111_{10}\,1$	$6.080_{10}{-1}$	$1.051_{10}\,1$	$6.283_{10}{-1}$
39	$1.249_{10}\,1$	$5.688_{10}{-1}$	$1.181_{10}\,1$	$5.898_{10}{-1}$	$1.118_{10}\,1$	$6.103_{10}{-1}$
40	$1.326_{10}\,1$	$5.507_{10}{-1}$	$1.255_{10}\,1$	$5.718_{10}{-1}$	$1.189_{10}\,1$	$5.924_{10}{-1}$
41	$1.406_{10}\,1$	$5.329_{10}{-1}$	$1.332_{10}\,1$	$5.539_{10}{-1}$	$1.262_{10}\,1$	$5.746_{10}{-1}$
42	$1.488_{10}\,1$	$5.155_{10}{-1}$	$1.411_{10}\,1$	$5.363_{10}{-1}$	$1.338_{10}\,1$	$5.569_{10}{-1}$
43	$1.573_{10}\,1$	$4.984_{10}{-1}$	$1.493_{10}\,1$	$5.190_{10}{-1}$	$1.417_{10}\,1$	$5.395_{10}{-1}$
44	$1.660_{10}\,1$	$4.819_{10}{-1}$	$1.578_{10}\,1$	$5.021_{10}{-1}$	$1.498_{10}\,1$	$5.224_{10}{-1}$
45	$1.749_{10}\,1$	$4.658_{10}{-1}$	$1.664_{10}\,1$	$4.857_{10}{-1}$	$1.582_{10}\,1$	$5.057_{10}{-1}$
46	$1.840_{10}\,1$	$4.504_{10}{-1}$	$1.753_{10}\,1$	$4.697_{10}{-1}$	$1.668_{10}\,1$	$4.894_{10}{-1}$
47	$1.932_{10}\,1$	$4.355_{10}{-1}$	$1.843_{10}\,1$	$4.543_{10}{-1}$	$1.756_{10}\,1$	$4.735_{10}{-1}$
48	$2.026_{10}\,1$	$4.212_{10}{-1}$	$1.935_{10}\,1$	$4.395_{10}{-1}$	$1.846_{10}\,1$	$4.582_{10}{-1}$
49	$2.120_{10}\,1$	$4.076_{10}{-1}$	$2.028_{10}\,1$	$4.252_{10}{-1}$	$1.937_{10}\,1$	$4.434_{10}{-1}$
50	$2.216_{10}\,1$	$3.945_{10}{-1}$	$2.122_{10}\,1$	$4.116_{10}{-1}$	$2.030_{10}\,1$	$4.291_{10}{-1}$
51	$2.313_{10}\,1$	$3.821_{10}{-1}$	$2.218_{10}\,1$	$3.985_{10}{-1}$	$2.124_{10}\,1$	$4.154_{10}{-1}$

n1	E[a] (n2 = 31)	D[a]/E[a] (n2 = 31)	E[a] (n2 = 32)	D[a]/E[a] (n2 = 32)	E[a] (n2 = 33)	D[a]/E[a] (n2 = 33)
1	1.059	$9.969_{10}-1$	1.057	$9.971_{10}-1$	1.056	$9.972_{10}-1$
2	1.121	$9.937_{10}-1$	1.117	$9.940_{10}-1$	1.114	$9.944_{10}-1$
3	1.186	$9.904_{10}-1$	1.181	$9.909_{10}-1$	1.175	$9.914_{10}-1$
4	1.255	$9.869_{10}-1$	1.247	$9.876_{10}-1$	1.240	$9.883_{10}-1$
5	1.328	$9.833_{10}-1$	1.318	$9.842_{10}-1$	1.308	$9.850_{10}-1$
6	1.405	$9.794_{10}-1$	1.392	$9.806_{10}-1$	1.380	$9.816_{10}-1$
7	1.487	$9.754_{10}-1$	1.471	$9.768_{10}-1$	1.456	$9.780_{10}-1$
8	1.573	$9.712_{10}-1$	1.554	$9.728_{10}-1$	1.536	$9.743_{10}-1$
9	1.665	$9.667_{10}-1$	1.642	$9.686_{10}-1$	1.621	$9.703_{10}-1$
10	1.763	$9.620_{10}-1$	1.736	$9.641_{10}-1$	1.711	$9.661_{10}-1$
11	1.866	$9.569_{10}-1$	1.835	$9.594_{10}-1$	1.806	$9.617_{10}-1$
12	1.977	$9.516_{10}-1$	1.940	$9.544_{10}-1$	1.906	$9.570_{10}-1$
13	2.094	$9.460_{10}-1$	2.052	$9.491_{10}-1$	2.014	$9.520_{10}-1$
14	2.219	$9.400_{10}-1$	2.171	$9.435_{10}-1$	2.127	$9.468_{10}-1$
15	2.353	$9.336_{10}-1$	2.298	$9.376_{10}-1$	2.248	$9.412_{10}-1$
16	2.495	$9.268_{10}-1$	2.434	$9.312_{10}-1$	2.377	$9.353_{10}-1$
17	2.647	$9.196_{10}-1$	2.578	$9.245_{10}-1$	2.514	$9.290_{10}-1$
18	2.810	$9.120_{10}-1$	2.732	$9.173_{10}-1$	2.660	$9.223_{10}-1$
19	2.985	$9.039_{10}-1$	2.897	$9.098_{10}-1$	2.816	$9.152_{10}-1$
20	3.171	$8.952_{10}-1$	3.073	$9.017_{10}-1$	2.983	$9.077_{10}-1$
21	3.371	$8.861_{10}-1$	3.261	$8.932_{10}-1$	3.161	$8.997_{10}-1$
22	3.585	$8.765_{10}-1$	3.463	$8.842_{10}-1$	3.351	$8.912_{10}-1$
23	3.815	$8.662_{10}-1$	3.679	$8.746_{10}-1$	3.554	$8.823_{10}-1$
24	4.062	$8.554_{10}-1$	3.910	$8.645_{10}-1$	3.771	$8.728_{10}-1$
25	4.326	$8.441_{10}-1$	4.158	$8.538_{10}-1$	4.004	$8.628_{10}-1$
26	4.609	$8.321_{10}-1$	4.423	$8.426_{10}-1$	4.253	$8.523_{10}-1$
27	4.913	$8.196_{10}-1$	4.708	$8.308_{10}-1$	4.520	$8.412_{10}-1$
28	5.238	$8.064_{10}-1$	5.012	$8.185_{10}-1$	4.805	$8.296_{10}-1$
29	5.587	$7.927_{10}-1$	5.338	$8.055_{10}-1$	5.110	$8.174_{10}-1$
30	5.960	$7.784_{10}-1$	5.687	$7.920_{10}-1$	5.436	$8.046_{10}-1$
31	6.358	$7.636_{10}-1$	6.060	$7.780_{10}-1$	5.785	$7.913_{10}-1$
32	6.784	$7.483_{10}-1$	6.458	$7.634_{10}-1$	6.158	$7.775_{10}-1$
33	7.237	$7.325_{10}-1$	6.882	$7.483_{10}-1$	6.555	$7.631_{10}-1$
34	7.720	$7.162_{10}-1$	7.334	$7.327_{10}-1$	6.979	$7.483_{10}-1$
35	8.232	$6.996_{10}-1$	7.815	$7.167_{10}-1$	7.430	$7.330_{10}-1$
36	8.775	$6.826_{10}-1$	8.325	$7.004_{10}-1$	7.909	$7.172_{10}-1$
37	9.349	$6.653_{10}-1$	8.865	$6.837_{10}-1$	8.417	$7.011_{10}-1$
38	9.954	$6.479_{10}-1$	9.436	$6.667_{10}-1$	8.955	$6.847_{10}-1$
39	$1.059_{10}\ 1$	$6.302_{10}-1$	$1.004_{10}\ 1$	$6.495_{10}-1$	9.522	$6.680_{10}-1$
40	$1.126_{10}\ 1$	$6.126_{10}-1$	$1.067_{10}\ 1$	$6.321_{10}-1$	$1.012_{10}\ 1$	$6.510_{10}-1$
41	$1.196_{10}\ 1$	$5.949_{10}-1$	$1.133_{10}\ 1$	$6.147_{10}-1$	$1.075_{10}\ 1$	$6.339_{10}-1$
42	$1.268_{10}\ 1$	$5.773_{10}-1$	$1.203_{10}\ 1$	$5.973_{10}-1$	$1.141_{10}\ 1$	$6.168_{10}-1$
43	$1.344_{10}\ 1$	$5.599_{10}-1$	$1.275_{10}\ 1$	$5.799_{10}-1$	$1.210_{10}\ 1$	$5.996_{10}-1$
44	$1.422_{10}\ 1$	$5.426_{10}-1$	$1.350_{10}\ 1$	$5.627_{10}-1$	$1.282_{10}\ 1$	$5.824_{10}-1$
45	$1.503_{10}\ 1$	$5.257_{10}-1$	$1.428_{10}\ 1$	$5.456_{10}-1$	$1.356_{10}\ 1$	$5.654_{10}-1$
46	$1.587_{10}\ 1$	$5.091_{10}-1$	$1.508_{10}\ 1$	$5.289_{10}-1$	$1.434_{10}\ 1$	$5.485_{10}-1$
47	$1.672_{10}\ 1$	$4.929_{10}-1$	$1.591_{10}\ 1$	$5.124_{10}-1$	$1.514_{10}\ 1$	$5.319_{10}-1$
48	$1.760_{10}\ 1$	$4.771_{10}-1$	$1.676_{10}\ 1$	$4.963_{10}-1$	$1.596_{10}\ 1$	$5.156_{10}-1$
49	$1.849_{10}\ 1$	$4.619_{10}-1$	$1.763_{10}\ 1$	$4.807_{10}-1$	$1.680_{10}\ 1$	$4.996_{10}-1$
50	$1.940_{10}\ 1$	$4.471_{10}-1$	$1.852_{10}\ 1$	$4.655_{10}-1$	$1.767_{10}\ 1$	$4.841_{10}-1$
51	$2.032_{10}\ 1$	$4.329_{10}-1$	$1.943_{10}\ 1$	$4.508_{10}-1$	$1.855_{10}\ 1$	$4.689_{10}-1$

	n2 = 34		n2 = 35		n2 = 36	
n1	E[a]	D[a]/E[a]	E[a]	D[a]/E[a]	E[a]	D[a]/E[a]
1	1.054	$9.974_{10}-1$	1.053	$9.975_{10}-1$	1.051	$9.976_{10}-1$
2	1.111	$9.946_{10}-1$	1.108	$9.949_{10}-1$	1.105	$9.952_{10}-1$
3	1.170	$9.918_{10}-1$	1.166	$9.922_{10}-1$	1.161	$9.926_{10}-1$
4	1.233	$9.889_{10}-1$	1.227	$9.894_{10}-1$	1.220	$9.899_{10}-1$
5	1.299	$9.858_{10}-1$	1.290	$9.865_{10}-1$	1.282	$9.872_{10}-1$
6	1.369	$9.826_{10}-1$	1.358	$9.835_{10}-1$	1.348	$9.843_{10}-1$
7	1.442	$9.792_{10}-1$	1.429	$9.803_{10}-1$	1.416	$9.813_{10}-1$
8	1.519	$9.756_{10}-1$	1.503	$9.769_{10}-1$	1.488	$9.781_{10}-1$
9	1.601	$9.719_{10}-1$	1.582	$9.734_{10}-1$	1.564	$9.747_{10}-1$
10	1.687	$9.680_{10}-1$	1.665	$9.696_{10}-1$	1.644	$9.712_{10}-1$
11	1.778	$9.638_{10}-1$	1.753	$9.657_{10}-1$	1.729	$9.675_{10}-1$
12	1.875	$9.594_{10}-1$	1.845	$9.616_{10}-1$	1.818	$9.636_{10}-1$
13	1.977	$9.547_{10}-1$	1.944	$9.572_{10}-1$	1.912	$9.594_{10}-1$
14	2.086	$9.498_{10}-1$	2.048	$9.525_{10}-1$	2.012	$9.551_{10}-1$
15	2.202	$9.445_{10}-1$	2.158	$9.476_{10}-1$	2.117	$9.504_{10}-1$
16	2.324	$9.390_{10}-1$	2.275	$9.424_{10}-1$	2.229	$9.455_{10}-1$
17	2.455	$9.331_{10}-1$	2.399	$9.368_{10}-1$	2.348	$9.403_{10}-1$
18	2.594	$9.268_{10}-1$	2.532	$9.310_{10}-1$	2.474	$9.348_{10}-1$
19	2.741	$9.202_{10}-1$	2.672	$9.247_{10}-1$	2.608	$9.290_{10}-1$
20	2.899	$9.131_{10}-1$	2.822	$9.181_{10}-1$	2.750	$9.228_{10}-1$
21	3.067	$9.056_{10}-1$	2.981	$9.111_{10}-1$	2.901	$9.162_{10}-1$
22	3.247	$8.977_{10}-1$	3.151	$9.037_{10}-1$	3.062	$9.092_{10}-1$
23	3.439	$8.894_{10}-1$	3.332	$8.959_{10}-1$	3.234	$9.019_{10}-1$
24	3.643	$8.805_{10}-1$	3.525	$8.876_{10}-1$	3.416	$8.941_{10}-1$
25	3.862	$8.711_{10}-1$	3.732	$8.788_{10}-1$	3.611	$8.859_{10}-1$
26	4.096	$8.613_{10}-1$	3.952	$8.695_{10}-1$	3.819	$8.772_{10}-1$
27	4.347	$8.509_{10}-1$	4.188	$8.597_{10}-1$	4.041	$8.680_{10}-1$
28	4.614	$8.399_{10}-1$	4.439	$8.495_{10}-1$	4.278	$8.583_{10}-1$
29	4.901	$8.284_{10}-1$	4.708	$8.387_{10}-1$	4.530	$8.481_{10}-1$
30	5.206	$8.164_{10}-1$	4.995	$8.273_{10}-1$	4.800	$8.374_{10}-1$
31	5.533	$8.038_{10}-1$	5.301	$8.154_{10}-1$	5.088	$8.262_{10}-1$
32	5.882	$7.907_{10}-1$	5.629	$8.030_{10}-1$	5.395	$8.145_{10}-1$
33	6.255	$7.771_{10}-1$	5.978	$7.901_{10}-1$	5.723	$8.023_{10}-1$
34	6.652	$7.629_{10}-1$	6.350	$7.767_{10}-1$	6.072	$7.895_{10}-1$
35	7.075	$7.483_{10}-1$	6.747	$7.627_{10}-1$	6.444	$7.763_{10}-1$
36	7.524	$7.332_{10}-1$	7.169	$7.483_{10}-1$	6.841	$7.625_{10}-1$
37	8.001	$7.177_{10}-1$	7.617	$7.335_{10}-1$	7.262	$7.483_{10}-1$
38	8.507	$7.019_{10}-1$	8.093	$7.182_{10}-1$	7.709	$7.337_{10}-1$
39	9.043	$6.857_{10}-1$	8.597	$7.026_{10}-1$	8.183	$7.187_{10}-1$
40	9.608	$6.692_{10}-1$	9.130	$6.866_{10}-1$	8.685	$7.033_{10}-1$
41	$1.020_{10}\ 1$	$6.525_{10}-1$	9.692	$6.704_{10}-1$	9.216	$6.875_{10}-1$
42	$1.083_{10}\ 1$	$6.357_{10}-1$	$1.028_{10}\ 1$	$6.540_{10}-1$	9.775	$6.715_{10}-1$
43	$1.148_{10}\ 1$	$6.187_{10}-1$	$1.091_{10}\ 1$	$6.373_{10}-1$	$1.036_{10}\ 1$	$6.553_{10}-1$
44	$1.217_{10}\ 1$	$6.018_{10}-1$	$1.156_{10}\ 1$	$6.206_{10}-1$	$1.098_{10}\ 1$	$6.389_{10}-1$
45	$1.288_{10}\ 1$	$5.848_{10}-1$	$1.224_{10}\ 1$	$6.039_{10}-1$	$1.163_{10}\ 1$	$6.225_{10}-1$
46	$1.362_{10}\ 1$	$5.680_{10}-1$	$1.295_{10}\ 1$	$5.871_{10}-1$	$1.231_{10}\ 1$	$6.059_{10}-1$
47	$1.439_{10}\ 1$	$5.513_{10}-1$	$1.368_{10}\ 1$	$5.705_{10}-1$	$1.301_{10}\ 1$	$5.894_{10}-1$
48	$1.519_{10}\ 1$	$5.349_{10}-1$	$1.445_{10}\ 1$	$5.540_{10}-1$	$1.375_{10}\ 1$	$5.729_{10}-1$
49	$1.600_{10}\ 1$	$5.187_{10}-1$	$1.524_{10}\ 1$	$5.377_{10}-1$	$1.450_{10}\ 1$	$5.566_{10}-1$
50	$1.685_{10}\ 1$	$5.028_{10}-1$	$1.605_{10}\ 1$	$5.217_{10}-1$	$1.529_{10}\ 1$	$5.405_{10}-1$
51	$1.771_{10}\ 1$	$4.874_{10}-1$	$1.689_{10}\ 1$	$5.059_{10}-1$	$1.610_{10}\ 1$	$5.246_{10}-1$

n1	n2 = 37 E[a]	n2 = 37 D[a]/E[a]	n2 = 38 E[a]	n2 = 38 D[a]/E[a]	n2 = 39 E[a]	n2 = 39 D[a]/E[a]
1	1.050	$9.977_{10}-1$	1.049	$9.978_{10}-1$	1.048	$9.979_{10}-1$
2	1.102	$9.954_{10}-1$	1.100	$9.956_{10}-1$	1.097	$9.958_{10}-1$
3	1.157	$9.930_{10}-1$	1.153	$9.933_{10}-1$	1.149	$9.936_{10}-1$
4	1.214	$9.904_{10}-1$	1.209	$9.909_{10}-1$	1.204	$9.913_{10}-1$
5	1.275	$9.878_{10}-1$	1.267	$9.884_{10}-1$	1.261	$9.889_{10}-1$
6	1.338	$9.850_{10}-1$	1.329	$9.858_{10}-1$	1.320	$9.864_{10}-1$
7	1.404	$9.822_{10}-1$	1.393	$9.830_{10}-1$	1.382	$9.838_{10}-1$
8	1.474	$9.792_{10}-1$	1.461	$9.802_{10}-1$	1.448	$9.811_{10}-1$
9	1.547	$9.760_{10}-1$	1.532	$9.772_{10}-1$	1.517	$9.782_{10}-1$
10	1.625	$9.727_{10}-1$	1.606	$9.740_{10}-1$	1.589	$9.752_{10}-1$
11	1.706	$9.691_{10}-1$	1.685	$9.707_{10}-1$	1.665	$9.721_{10}-1$
12	1.792	$9.654_{10}-1$	1.767	$9.672_{10}-1$	1.744	$9.688_{10}-1$
13	1.882	$9.615_{10}-1$	1.855	$9.635_{10}-1$	1.829	$9.653_{10}-1$
14	1.978	$9.574_{10}-1$	1.947	$9.596_{10}-1$	1.917	$9.616_{10}-1$
15	2.079	$9.530_{10}-1$	2.044	$9.555_{10}-1$	2.010	$9.577_{10}-1$
16	2.187	$9.484_{10}-1$	2.147	$9.511_{10}-1$	2.109	$9.536_{10}-1$
17	2.300	$9.435_{10}-1$	2.255	$9.465_{10}-1$	2.213	$9.493_{10}-1$
18	2.420	$9.383_{10}-1$	2.370	$9.416_{10}-1$	2.323	$9.447_{10}-1$
19	2.548	$9.329_{10}-1$	2.492	$9.365_{10}-1$	2.440	$9.398_{10}-1$
20	2.683	$9.270_{10}-1$	2.621	$9.310_{10}-1$	2.563	$9.346_{10}-1$
21	2.827	$9.209_{10}-1$	2.758	$9.252_{10}-1$	2.693	$9.292_{10}-1$
22	2.980	$9.143_{10}-1$	2.903	$9.191_{10}-1$	2.832	$9.234_{10}-1$
23	3.142	$9.074_{10}-1$	3.058	$9.126_{10}-1$	2.979	$9.173_{10}-1$
24	3.315	$9.001_{10}-1$	3.222	$9.057_{10}-1$	3.135	$9.108_{10}-1$
25	3.500	$8.924_{10}-1$	3.396	$8.984_{10}-1$	3.300	$9.040_{10}-1$
26	3.696	$8.842_{10}-1$	3.582	$8.908_{10}-1$	3.476	$8.968_{10}-1$
27	3.905	$8.756_{10}-1$	3.780	$8.826_{10}-1$	3.664	$8.892_{10}-1$
28	4.129	$8.665_{10}-1$	3.991	$8.741_{10}-1$	3.863	$8.811_{10}-1$
29	4.367	$8.569_{10}-1$	4.215	$8.651_{10}-1$	4.075	$8.727_{10}-1$
30	4.620	$8.469_{10}-1$	4.454	$8.556_{10}-1$	4.301	$8.637_{10}-1$
31	4.891	$8.363_{10}-1$	4.709	$8.456_{10}-1$	4.541	$8.543_{10}-1$
32	5.180	$8.252_{10}-1$	4.981	$8.352_{10}-1$	4.797	$8.445_{10}-1$
33	5.488	$8.136_{10}-1$	5.270	$8.242_{10}-1$	5.070	$8.341_{10}-1$
34	5.816	$8.016_{10}-1$	5.579	$8.128_{10}-1$	5.360	$8.233_{10}-1$
35	6.165	$7.890_{10}-1$	5.907	$8.009_{10}-1$	5.669	$8.120_{10}-1$
36	6.537	$7.759_{10}-1$	6.257	$7.884_{10}-1$	5.998	$8.002_{10}-1$
37	6.933	$7.623_{10}-1$	6.629	$7.755_{10}-1$	6.348	$7.879_{10}-1$
38	7.353	$7.483_{10}-1$	7.024	$7.622_{10}-1$	6.719	$7.752_{10}-1$
39	7.800	$7.339_{10}-1$	7.444	$7.484_{10}-1$	7.114	$7.620_{10}-1$
40	8.272	$7.191_{10}-1$	7.889	$7.341_{10}-1$	7.533	$7.484_{10}-1$
41	8.773	$7.039_{10}-1$	8.360	$7.195_{10}-1$	7.977	$7.343_{10}-1$
42	9.301	$6.884_{10}-1$	8.859	$7.045_{10}-1$	8.447	$7.199_{10}-1$
43	9.858	$6.726_{10}-1$	9.385	$6.893_{10}-1$	8.944	$7.052_{10}-1$
44	$1.044_{10}\ 1$	$6.566_{10}-1$	9.940	$6.737_{10}-1$	9.468	$6.901_{10}-1$
45	$1.106_{10}\ 1$	$6.405_{10}-1$	$1.052_{10}\ 1$	$6.579_{10}-1$	$1.002_{10}\ 1$	$6.747_{10}-1$
46	$1.170_{10}\ 1$	$6.242_{10}-1$	$1.113_{10}\ 1$	$6.420_{10}-1$	$1.060_{10}\ 1$	$6.591_{10}-1$
47	$1.238_{10}\ 1$	$6.079_{10}-1$	$1.178_{10}\ 1$	$6.259_{10}-1$	$1.121_{10}\ 1$	$6.434_{10}-1$
48	$1.308_{10}\ 1$	$5.915_{10}-1$	$1.244_{10}\ 1$	$6.098_{10}-1$	$1.185_{10}\ 1$	$6.275_{10}-1$
49	$1.381_{10}\ 1$	$5.753_{10}-1$	$1.314_{10}\ 1$	$5.936_{10}-1$	$1.251_{10}\ 1$	$6.116_{10}-1$
50	$1.456_{10}\ 1$	$5.591_{10}-1$	$1.387_{10}\ 1$	$5.775_{10}-1$	$1.321_{10}\ 1$	$5.956_{10}-1$
51	$1.534_{10}\ 1$	$5.431_{10}-1$	$1.462_{10}\ 1$	$5.615_{10}-1$	$1.393_{10}\ 1$	$5.797_{10}-1$

	n2 = 40		n2 = 41		n2 = 42	
n1	E[a]	D[a]/E[a]	E[a]	D[a]/E[a]	E[a]	D[a]/E[a]
1	1.047	$9.980_{10}-1$	1.045	$9.981_{10}-1$	1.044	$9.982_{10}-1$
2	1.095	$9.960_{10}-1$	1.093	$9.962_{10}-1$	1.091	$9.963_{10}-1$
3	1.146	$9.939_{10}-1$	1.142	$9.941_{10}-1$	1.139	$9.944_{10}-1$
4	1.199	$9.917_{10}-1$	1.194	$9.920_{10}-1$	1.189	$9.924_{10}-1$
5	1.254	$9.894_{10}-1$	1.248	$9.899_{10}-1$	1.242	$9.903_{10}-1$
6	1.312	$9.870_{10}-1$	1.304	$9.876_{10}-1$	1.297	$9.881_{10}-1$
7	1.372	$9.846_{10}-1$	1.363	$9.852_{10}-1$	1.354	$9.859_{10}-1$
8	1.436	$9.820_{10}-1$	1.425	$9.828_{10}-1$	1.414	$9.835_{10}-1$
9	1.503	$9.793_{10}-1$	1.489	$9.802_{10}-1$	1.476	$9.811_{10}-1$
10	1.572	$9.764_{10}-1$	1.557	$9.775_{10}-1$	1.542	$9.785_{10}-1$
11	1.646	$9.734_{10}-1$	1.628	$9.747_{10}-1$	1.611	$9.758_{10}-1$
12	1.723	$9.703_{10}-1$	1.702	$9.717_{10}-1$	1.683	$9.730_{10}-1$
13	1.804	$9.670_{10}-1$	1.781	$9.685_{10}-1$	1.759	$9.700_{10}-1$
14	1.889	$9.635_{10}-1$	1.867	$9.652_{10}-1$	1.838	$9.668_{10}-1$
15	1.979	$9.598_{10}-1$	1.949	$9.617_{10}-1$	1.921	$9.635_{10}-1$
16	2.074	$9.559_{10}-1$	2.041	$9.581_{10}-1$	2.009	$9.601_{10}-1$
17	2.174	$9.518_{10}-1$	2.137	$9.542_{10}-1$	2.102	$9.564_{10}-1$
18	2.279	$9.475_{10}-1$	2.238	$9.501_{10}-1$	2.199	$9.525_{10}-1$
19	2.391	$9.429_{10}-1$	2.345	$9.457_{10}-1$	2.302	$9.484_{10}-1$
20	2.509	$9.380_{10}-1$	2.458	$9.412_{10}-1$	2.410	$9.441_{10}-1$
21	2.633	$9.329_{10}-1$	2.577	$9.363_{10}-1$	2.524	$9.395_{10}-1$
22	2.765	$9.275_{10}-1$	2.703	$9.312_{10}-1$	2.645	$9.347_{10}-1$
23	2.905	$9.217_{10}-1$	2.837	$9.258_{10}-1$	2.772	$9.296_{10}-1$
24	3.054	$9.156_{10}-1$	2.978	$9.201_{10}-1$	2.907	$9.242_{10}-1$
25	3.211	$9.092_{10}-1$	3.128	$9.140_{10}-1$	3.050	$9.185_{10}-1$
26	3.378	$9.024_{10}-1$	3.287	$9.076_{10}-1$	3.201	$9.124_{10}-1$
27	3.556	$8.952_{10}-1$	3.455	$9.009_{10}-1$	3.362	$9.061_{10}-1$
28	3.744	$8.877_{10}-1$	3.634	$8.937_{10}-1$	3.531	$8.994_{10}-1$
29	3.945	$8.797_{10}-1$	3.824	$8.862_{10}-1$	3.712	$8.923_{10}-1$
30	4.158	$8.713_{10}-1$	4.026	$8.783_{10}-1$	3.903	$8.848_{10}-1$
31	4.385	$8.624_{10}-1$	4.241	$8.699_{10}-1$	4.106	$8.769_{10}-1$
32	4.627	$8.531_{10}-1$	4.469	$8.612_{10}-1$	4.322	$8.686_{10}-1$
33	4.884	$8.433_{10}-1$	4.711	$8.519_{10}-1$	4.552	$8.599_{10}-1$
34	5.157	$8.331_{10}-1$	4.969	$8.423_{10}-1$	4.795	$8.508_{10}-1$
35	5.448	$8.224_{10}-1$	5.244	$8.321_{10}-1$	5.054	$8.412_{10}-1$
36	5.758	$8.112_{10}-1$	5.535	$8.215_{10}-1$	5.329	$8.312_{10}-1$
37	6.087	$7.996_{10}-1$	5.846	$8.105_{10}-1$	5.622	$8.207_{10}-1$
38	6.437	$7.874_{10}-1$	6.175	$7.990_{10}-1$	5.932	$8.098_{10}-1$
39	6.809	$7.749_{10}-1$	6.525	$7.870_{10}-1$	6.262	$7.984_{10}-1$
40	7.203	$7.618_{10}-1$	6.897	$7.746_{10}-1$	6.612	$7.865_{10}-1$
41	7.622	$7.484_{10}-1$	7.291	$7.617_{10}-1$	6.984	$7.743_{10}-1$
42	8.065	$7.345_{10}-1$	7.709	$7.484_{10}-1$	7.378	$7.615_{10}-1$
43	8.533	$7.203_{10}-1$	8.151	$7.347_{10}-1$	7.795	$7.484_{10}-1$
44	9.029	$7.057_{10}-1$	8.618	$7.207_{10}-1$	8.236	$7.349_{10}-1$
45	9.551	$6.909_{10}-1$	9.112	$7.063_{10}-1$	8.702	$7.211_{10}-1$
46	$1.010_{10}\ 1$	$6.757_{10}-1$	9.632	$6.916_{10}-1$	9.195	$7.069_{10}-1$
47	$1.068_{10}\ 1$	$6.603_{10}-1$	$1.018_{10}\ 1$	$6.767_{10}-1$	9.713	$6.923_{10}-1$
48	$1.128_{10}\ 1$	$6.448_{10}-1$	$1.076_{10}\ 1$	$6.615_{10}-1$	$1.026_{10}\ 1$	$6.776_{10}-1$
49	$1.192_{10}\ 1$	$6.291_{10}-1$	$1.136_{10}\ 1$	$6.461_{10}-1$	$1.083_{10}\ 1$	$6.626_{10}-1$
50	$1.258_{10}\ 1$	$6.133_{10}-1$	$1.199_{10}\ 1$	$6.306_{10}-1$	$1.143_{10}\ 1$	$6.474_{10}-1$
51	$1.327_{10}\ 1$	$5.975_{10}-1$	$1.265_{10}\ 1$	$6.150_{10}-1$	$1.206_{10}\ 1$	$6.321_{10}-1$

n1	n2 = 43 E[a]	n2 = 43 D[a]/E[a]	n2 = 44 E[a]	n2 = 44 D[a]/E[a]	n2 = 45 E[a]	n2 = 45 D[a]/E[a]
1	1.043	$9.983_{10}-1$	1.043	$9.983_{10}-1$	1.042	$9.984_{10}-1$
2	1.089	$9.965_{10}-1$	1.087	$9.966_{10}-1$	1.085	$9.967_{10}-1$
3	1.136	$9.946_{10}-1$	1.133	$9.948_{10}-1$	1.130	$9.950_{10}-1$
4	1.185	$9.927_{10}-1$	1.181	$9.930_{10}-1$	1.177	$9.933_{10}-1$
5	1.236	$9.907_{10}-1$	1.231	$9.911_{10}-1$	1.226	$9.915_{10}-1$
6	1.290	$9.887_{10}-1$	1.283	$9.891_{10}-1$	1.276	$9.896_{10}-1$
7	1.345	$9.865_{10}-1$	1.337	$9.871_{10}-1$	1.329	$9.876_{10}-1$
8	1.403	$9.843_{10}-1$	1.394	$9.849_{10}-1$	1.384	$9.855_{10}-1$
9	1.464	$9.819_{10}-1$	1.453	$9.827_{10}-1$	1.442	$9.834_{10}-1$
10	1.528	$9.795_{10}-1$	1.515	$9.803_{10}-1$	1.502	$9.812_{10}-1$
11	1.595	$9.769_{10}-1$	1.579	$9.779_{10}-1$	1.565	$9.788_{10}-1$
12	1.665	$9.742_{10}-1$	1.647	$9.753_{10}-1$	1.631	$9.764_{10}-1$
13	1.738	$9.713_{10}-1$	1.718	$9.726_{10}-1$	1.699	$9.738_{10}-1$
14	1.814	$9.684_{10}-1$	1.792	$9.698_{10}-1$	1.771	$9.711_{10}-1$
15	1.895	$9.652_{10}-1$	1.870	$9.668_{10}-1$	1.846	$9.683_{10}-1$
16	1.980	$9.619_{10}-1$	1.952	$9.637_{10}-1$	1.925	$9.653_{10}-1$
17	2.069	$9.584_{10}-1$	2.038	$9.604_{10}-1$	2.008	$9.621_{10}-1$
18	2.162	$9.548_{10}-1$	2.128	$9.569_{10}-1$	2.095	$9.588_{10}-1$
19	2.261	$9.509_{10}-1$	2.223	$9.532_{10}-1$	2.187	$9.554_{10}-1$
20	2.365	$9.468_{10}-1$	2.323	$9.493_{10}-1$	2.283	$9.517_{10}-1$
21	2.474	$9.425_{10}-1$	2.428	$9.452_{10}-1$	2.384	$9.478_{10}-1$
22	2.590	$9.379_{10}-1$	2.539	$9.409_{10}-1$	2.490	$9.437_{10}-1$
23	2.712	$9.331_{10}-1$	2.655	$9.364_{10}-1$	2.602	$9.394_{10}-1$
24	2.841	$9.280_{10}-1$	2.779	$9.316_{10}-1$	2.720	$9.349_{10}-1$
25	2.977	$9.226_{10}-1$	2.909	$9.265_{10}-1$	2.845	$9.301_{10}-1$
26	3.121	$9.169_{10}-1$	3.047	$9.211_{10}-1$	2.977	$9.250_{10}-1$
27	3.274	$9.109_{10}-1$	3.192	$9.154_{10}-1$	3.116	$9.197_{10}-1$
28	3.436	$9.046_{10}-1$	3.346	$9.095_{10}-1$	3.263	$9.140_{10}-1$
29	3.607	$8.979_{10}-1$	3.509	$9.032_{10}-1$	3.418	$9.081_{10}-1$
30	3.789	$8.909_{10}-1$	3.682	$8.965_{10}-1$	3.582	$9.018_{10}-1$
31	3.981	$8.834_{10}-1$	3.865	$8.895_{10}-1$	3.756	$8.952_{10}-1$
32	4.186	$8.756_{10}-1$	4.059	$8.821_{10}-1$	3.940	$8.882_{10}-1$
33	4.403	$8.674_{10}-1$	4.265	$8.744_{10}-1$	4.136	$8.809_{10}-1$
34	4.633	$8.588_{10}-1$	4.483	$8.662_{10}-1$	4.342	$8.732_{10}-1$
35	4.878	$8.497_{10}-1$	4.714	$8.576_{10}-1$	4.562	$8.651_{10}-1$
36	5.138	$8.402_{10}-1$	4.960	$8.487_{10}-1$	4.794	$8.566_{10}-1$
37	5.414	$8.303_{10}-1$	5.221	$8.393_{10}-1$	5.041	$8.476_{10}-1$
38	5.707	$8.199_{10}-1$	5.498	$8.294_{10}-1$	5.303	$8.383_{10}-1$
39	6.018	$8.091_{10}-1$	5.791	$8.191_{10}-1$	5.580	$8.286_{10}-1$
40	6.348	$7.978_{10}-1$	6.103	$8.084_{10}-1$	5.875	$8.184_{10}-1$
41	6.699	$7.861_{10}-1$	6.434	$7.973_{10}-1$	6.187	$8.078_{10}-1$
42	7.070	$7.740_{10}-1$	6.784	$7.857_{10}-1$	6.518	$7.967_{10}-1$
43	7.464	$7.614_{10}-1$	7.155	$7.737_{10}-1$	6.868	$7.853_{10}-1$
44	7.880	$7.485_{10}-1$	7.548	$7.613_{10}-1$	7.239	$7.734_{10}-1$
45	8.320	$7.351_{10}-1$	7.964	$7.485_{10}-1$	7.632	$7.612_{10}-1$
46	8.786	$7.214_{10}-1$	8.404	$7.353_{10}-1$	8.048	$7.485_{10}-1$
47	9.276	$7.074_{10}-1$	8.868	$7.218_{10}-1$	8.486	$7.355_{10}-1$
48	9.793	$6.930_{10}-1$	9.357	$7.079_{10}-1$	8.949	$7.221_{10}-1$
49	1.034_{10} 1	$6.785_{10}-1$	9.872	$6.937_{10}-1$	9.437	$7.084_{10}-1$
50	1.091_{10} 1	$6.636_{10}-1$	1.041_{10} 1	$6.793_{10}-1$	9.950	$6.944_{10}-1$
51	1.150_{10} 1	$6.486_{10}-1$	1.098_{10} 1	$6.647_{10}-1$	1.049_{10} 1	$6.801_{10}-1$

	n2 = 46		n2 = 47		n2 = 48	
n1	E[a]	D[a]/E[a]	E[a]	D[a]/E[a]	E[a]	D[a]/E[a]
1	1.041	$9.985_{10}-1$	1.040	$9.985_{10}-1$	1.039	$9.986_{10}-1$
2	1.083	$9.969_{10}-1$	1.082	$9.970_{10}-1$	1.080	$9.971_{10}-1$
3	1.127	$9.952_{10}-1$	1.125	$9.954_{10}-1$	1.122	$9.956_{10}-1$
4	1.173	$9.936_{10}-1$	1.169	$9.938_{10}-1$	1.166	$9.940_{10}-1$
5	1.221	$9.918_{10}-1$	1.216	$9.921_{10}-1$	1.211	$9.924_{10}-1$
6	1.270	$9.900_{10}-1$	1.264	$9.904_{10}-1$	1.259	$9.907_{10}-1$
7	1.322	$9.881_{10}-1$	1.315	$9.886_{10}-1$	1.308	$9.890_{10}-1$
8	1.375	$9.861_{10}-1$	1.367	$9.867_{10}-1$	1.359	$9.872_{10}-1$
9	1.432	$9.841_{10}-1$	1.422	$9.847_{10}-1$	1.412	$9.853_{10}-1$
10	1.490	$9.819_{10}-1$	1.478	$9.827_{10}-1$	1.467	$9.834_{10}-1$
11	1.551	$9.797_{10}-1$	1.538	$9.805_{10}-1$	1.525	$9.813_{10}-1$
12	1.615	$9.774_{10}-1$	1.600	$9.783_{10}-1$	1.585	$9.792_{10}-1$
13	1.681	$9.749_{10}-1$	1.664	$9.760_{10}-1$	1.648	$9.769_{10}-1$
14	1.751	$9.723_{10}-1$	1.732	$9.735_{10}-1$	1.714	$9.746_{10}-1$
15	1.824	$9.696_{10}-1$	1.803	$9.709_{10}-1$	1.782	$9.721_{10}-1$
16	1.900	$9.668_{10}-1$	1.877	$9.682_{10}-1$	1.854	$9.696_{10}-1$
17	1.980	$9.638_{10}-1$	1.954	$9.654_{10}-1$	1.929	$9.669_{10}-1$
18	2.064	$9.607_{10}-1$	2.035	$9.624_{10}-1$	2.007	$9.640_{10}-1$
19	2.153	$9.574_{10}-1$	2.120	$9.593_{10}-1$	2.090	$9.610_{10}-1$
20	2.245	$9.539_{10}-1$	2.210	$9.560_{10}-1$	2.176	$9.579_{10}-1$
21	2.342	$9.502_{10}-1$	2.303	$9.525_{10}-1$	2.266	$9.546_{10}-1$
22	2.445	$9.464_{10}-1$	2.402	$9.488_{10}-1$	2.361	$9.511_{10}-1$
23	2.552	$9.423_{10}-1$	2.505	$9.449_{10}-1$	2.460	$9.474_{10}-1$
24	2.666	$9.380_{10}-1$	2.614	$9.409_{10}-1$	2.565	$9.436_{10}-1$
25	2.785	$9.334_{10}-1$	2.728	$9.366_{10}-1$	2.675	$9.395_{10}-1$
26	2.911	$9.286_{10}-1$	2.849	$9.320_{10}-1$	2.791	$9.352_{10}-1$
27	3.044	$9.236_{10}-1$	2.976	$9.272_{10}-1$	2.913	$9.307_{10}-1$
28	3.184	$9.182_{10}-1$	3.110	$9.222_{10}-1$	3.041	$9.259_{10}-1$
29	3.332	$9.126_{10}-1$	3.252	$9.169_{10}-1$	3.177	$9.209_{10}-1$
30	3.489	$9.067_{10}-1$	3.401	$9.113_{10}-1$	3.319	$9.156_{10}-1$
31	3.654	$9.004_{10}-1$	3.559	$9.054_{10}-1$	3.470	$9.100_{10}-1$
32	3.830	$8.939_{10}-1$	3.726	$8.991_{10}-1$	3.629	$9.041_{10}-1$
33	4.015	$8.869_{10}-1$	3.903	$8.926_{10}-1$	3.797	$8.979_{10}-1$
34	4.212	$8.796_{10}-1$	4.089	$8.857_{10}-1$	3.975	$8.914_{10}-1$
35	4.420	$8.720_{10}-1$	4.287	$8.785_{10}-1$	4.163	$8.845_{10}-1$
36	4.640	$8.639_{10}-1$	4.496	$8.708_{10}-1$	4.361	$8.773_{10}-1$
37	4.874	$8.555_{10}-1$	4.718	$8.629_{10}-1$	4.572	$8.697_{10}-1$
38	5.121	$8.467_{10}-1$	4.952	$8.545_{10}-1$	4.794	$8.618_{10}-1$
39	5.384	$8.374_{10}-1$	5.201	$8.457_{10}-1$	5.030	$8.535_{10}-1$
40	5.662	$8.278_{10}-1$	5.464	$8.365_{10}-1$	5.279	$8.448_{10}-1$
41	5.957	$8.177_{10}-1$	5.743	$8.270_{10}-1$	5.544	$8.357_{10}-1$
42	6.270	$8.072_{10}-1$	6.039	$8.170_{10}-1$	5.823	$8.262_{10}-1$
43	6.601	$7.962_{10}-1$	6.352	$8.066_{10}-1$	6.120	$8.163_{10}-1$
44	6.952	$7.849_{10}-1$	6.683	$7.958_{10}-1$	6.433	$8.060_{10}-1$
45	7.323	$7.732_{10}-1$	7.034	$7.845_{10}-1$	6.765	$7.953_{10}-1$
46	7.715	$7.610_{10}-1$	7.405	$7.729_{10}-1$	7.116	$7.842_{10}-1$
47	8.130	$7.485_{10}-1$	7.797	$7.609_{10}-1$	7.487	$7.727_{10}-1$
48	8.568	$7.356_{10}-1$	8.212	$7.485_{10}-1$	7.879	$7.608_{10}-1$
49	9.030	$7.224_{10}-1$	8.649	$7.358_{10}-1$	8.292	$7.486_{10}-1$
50	9.516	$7.089_{10}-1$	9.110	$7.227_{10}-1$	8.729	$7.360_{10}-1$
51	$1.003_{10}\ 1$	$6.950_{10}-1$	9.595	$7.093_{10}-1$	9.189	$7.230_{10}-1$

	n2 = 49		n2 = 50		n2 = 51	
n1	E[a]	D[a]/E[a]	E[a]	D[a]/E[a]	E[a]	D[a]/E[a]
1	1.038	$9.986_{10}-1$	1.038	$9.987_{10}-1$	1.037	$9.987_{10}-1$
2	1.078	$9.972_{10}-1$	1.077	$9.973_{10}-1$	1.075	$9.974_{10}-1$
3	1.120	$9.958_{10}-1$	1.117	$9.959_{10}-1$	1.115	$9.961_{10}-1$
4	1.163	$9.943_{10}-1$	1.159	$9.945_{10}-1$	1.156	$9.947_{10}-1$
5	1.207	$9.927_{10}-1$	1.203	$9.930_{10}-1$	1.199	$9.932_{10}-1$
6	1.253	$9.911_{10}-1$	1.248	$9.914_{10}-1$	1.243	$9.917_{10}-1$
7	1.301	$9.894_{10}-1$	1.295	$9.898_{10}-1$	1.289	$9.902_{10}-1$
8	1.351	$9.877_{10}-1$	1.344	$9.882_{10}-1$	1.336	$9.886_{10}-1$
9	1.403	$9.859_{10}-1$	1.394	$9.864_{10}-1$	1.386	$9.869_{10}-1$
10	1.457	$9.840_{10}-1$	1.447	$9.846_{10}-1$	1.437	$9.852_{10}-1$
11	1.513	$9.821_{10}-1$	1.502	$9.828_{10}-1$	1.491	$9.834_{10}-1$
12	1.572	$9.800_{10}-1$	1.559	$9.808_{10}-1$	1.546	$9.815_{10}-1$
13	1.633	$9.779_{10}-1$	1.618	$9.787_{10}-1$	1.604	$9.796_{10}-1$
14	1.697	$9.756_{10}-1$	1.680	$9.766_{10}-1$	1.664	$9.775_{10}-1$
15	1.763	$9.733_{10}-1$	1.745	$9.744_{10}-1$	1.727	$9.754_{10}-1$
16	1.833	$9.708_{10}-1$	1.812	$9.720_{10}-1$	1.793	$9.731_{10}-1$
17	1.905	$9.683_{10}-1$	1.883	$9.696_{10}-1$	1.861	$9.708_{10}-1$
18	1.981	$9.655_{10}-1$	1.956	$9.670_{10}-1$	1.932	$9.683_{10}-1$
19	2.061	$9.627_{10}-1$	2.033	$9.643_{10}-1$	2.007	$9.657_{10}-1$
20	2.144	$9.597_{10}-1$	2.113	$9.614_{10}-1$	2.085	$9.630_{10}-1$
21	2.231	$9.566_{10}-1$	2.198	$9.584_{10}-1$	2.166	$9.602_{10}-1$
22	2.322	$9.533_{10}-1$	2.286	$9.553_{10}-1$	2.251	$9.572_{10}-1$
23	2.418	$9.498_{10}-1$	2.378	$9.520_{10}-1$	2.341	$9.540_{10}-1$
24	2.519	$9.461_{10}-1$	2.475	$9.485_{10}-1$	2.434	$9.507_{10}-1$
25	2.625	$9.422_{10}-1$	2.577	$9.448_{10}-1$	2.532	$9.472_{10}-1$
26	2.736	$9.382_{10}-1$	2.684	$9.409_{10}-1$	2.635	$9.435_{10}-1$
27	2.853	$9.339_{10}-1$	2.796	$9.369_{10}-1$	2.743	$9.397_{10}-1$
28	2.976	$9.293_{10}-1$	2.914	$9.326_{10}-1$	2.856	$9.356_{10}-1$
29	3.106	$9.246_{10}-1$	3.039	$9.281_{10}-1$	2.976	$9.313_{10}-1$
30	3.242	$9.196_{10}-1$	3.170	$9.233_{10}-1$	3.101	$9.268_{10}-1$
31	3.386	$9.143_{10}-1$	3.307	$9.183_{10}-1$	3.233	$9.221_{10}-1$
32	3.538	$9.087_{10}-1$	3.452	$9.130_{10}-1$	3.372	$9.171_{10}-1$
33	3.698	$9.028_{10}-1$	3.606	$9.075_{10}-1$	3.518	$9.118_{10}-1$
34	3.868	$8.967_{10}-1$	3.767	$9.016_{10}-1$	3.673	$9.063_{10}-1$
35	4.046	$8.902_{10}-1$	3.938	$8.955_{10}-1$	3.835	$9.004_{10}-1$
36	4.236	$8.834_{10}-1$	4.118	$8.890_{10}-1$	4.007	$8.943_{10}-1$
37	4.435	$8.762_{10}-1$	4.308	$8.822_{10}-1$	4.188	$8.879_{10}-1$
38	4.647	$8.687_{10}-1$	4.509	$8.751_{10}-1$	4.379	$8.811_{10}-1$
39	4.870	$8.608_{10}-1$	4.721	$8.676_{10}-1$	4.581	$8.741_{10}-1$
40	5.107	$8.525_{10}-1$	4.946	$8.598_{10}-1$	4.795	$8.666_{10}-1$
41	5.357	$8.439_{10}-1$	5.183	$8.516_{10}-1$	5.020	$8.589_{10}-1$
42	5.622	$8.349_{10}-1$	5.435	$8.430_{10}-1$	5.259	$8.507_{10}-1$
43	5.903	$8.255_{10}-1$	5.700	$8.341_{10}-1$	5.511	$8.422_{10}-1$
44	6.199	$8.157_{10}-1$	5.981	$8.247_{10}-1$	5.777	$8.333_{10}-1$
45	6.513	$8.054_{10}-1$	6.279	$8.150_{10}-1$	6.059	$8.241_{10}-1$
46	6.845	$7.948_{10}-1$	6.593	$8.049_{10}-1$	6.357	$8.144_{10}-1$
47	7.196	$7.838_{10}-1$	6.925	$7.944_{10}-1$	6.672	$8.044_{10}-1$
48	7.567	$7.725_{10}-1$	7.276	$7.835_{10}-1$	7.004	$7.940_{10}-1$
49	7.959	$7.607_{10}-1$	7.647	$7.722_{10}-1$	7.355	$7.832_{10}-1$
50	8.372	$7.486_{10}-1$	8.039	$7.606_{10}-1$	7.726	$7.720_{10}-1$
51	8.808	$7.361_{10}-1$	8.451	$7.486_{10}-1$	8.117	$7.605_{10}-1$

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