Modelling of Uncertainties and Price Elastic Demands in Energy-Environment Planning for India*

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Abstract

This paper describes two variants of the Indian MARKAL model, a long-term technology oriented optimisation model for energy-environment planning for India. The first variant uses stochastic programming to include future uncertainties in the analysis. Details of model formulation, results and sensitivity analysis are described here. The second variant uses an innovative approach to simulate price sensitive demands within a linear formulation. The analysis incorporating future uncertainties suggests that it is prudent to reduce carbon emission in anticipation of a global regime in future. Modelling with price elastic demands estimates up to ten percent reduction in carbon emission due to reduced demands, under a severe carbon tax.

1. Introduction

Climate change has made environment an integral part of long-term energy planning. Mathematical models are frequently used for policy analysis in this complex system. Energy-environment modelling has to deal with a number of future uncertainties. Some arising out of the complexities in the climate change process, and others due to the long planning horizons that must be considered. Usually a scenario approach is taken and different possible futures are examined independent of one another. While this approach is useful for broad long-term conclusions, it has severe limitations for immediate operational decisions, as different scenario results may recommend different actions. This paper describes an application of stochastic programming to incorporate future uncertainties in long-term energy-environment modelling.

Detailed technology evaluation models like the MARKAL model [1] often ignore the macroeconomic impacts of implementing carbon mitigation, mainly in terms of demand reductions induced by increased energy prices. This paper also includes analysis based on an innovative approach to simulate price sensitive demands within a linear formulation.

The modelling innovations described in this paper have been done on the Indian MARKAL (IM) model [2,3,4]. The MARKAL approach is briefly summarised in section 2. Section 3 has a detailed description of the formulation and results of the stochastic Indian MARKAL (stochastic IM). The elastic Indian MARKAL (elastic IM) is described in section 4. Section 5 concludes this paper.

2. The MARKAL Approach

MARKAL is a large scale technology oriented activity analysis model, integrating the supply and end-use sectors of an economy, with emphasis on the description of energy related sub-sectors. The model has nine time periods of five years each (thus covering the 45 year span from 1993 to 2038), and utilises three variables for each technology represented, i.e. the

investment, the capacity, and the level of activity of the technology, at each time period (at period 1, the actual installed capacities of all technologies are imposed, thus constraining MARKAL to exactly represent the real system being modelled). MARKAL computes a dynamic, partial equilibrium on energy markets by minimising a single objective function which is the system's discounted total cost (the equilibrium is partial rather than general, since MARKAL does not include links with other macroeconomic variables, such as aggregate savings, consumption etc.). The system's cost includes investment and operations and maintenance costs for all technologies, plus procurement costs for all imported fuels, minus the revenue from exported fuels, minus the salvage value of all residual technologies at the end of the horizon. The model satisfies all important constraints of an energy system, such as flow conservation, satisfaction of demands, conservation of investments, peak-electricity constraints, capacity limits, and many others. In addition, MARKAL allows the optional accounting and/or constraining of emissions of pollutants from all technologies present in the model, by means of emission coefficients and of special constraints (alternatively, one may impose emission taxes rather than constraints). In order to simultaneously respect these constraints and minimise system cost, MARKAL uses optimisation (Linear Programming).

3. Stochastic Indian MARKAL

Stochastic Models in Energy Planning

In the context of energy-environment systems, stochastic modelling has been extensively used to study the energy resource extraction process [5,6,7,8] and optimizing the electricity generation process [9,10,11,12]. Studies of socio-economic impacts of the uncertain outcomes of global warming have also used stochastic models [13,14,15]. A model for stochastic power generation planning problem was presented with a simple application in Louveaux and Smeers [16].

A two-step model for robustness analysis in energy planning was suggested in Wene [17]. A comprehensive description of the method and its application can be found in Larsson and Wene [18] and Larsson [19]. This method provided for assessing the efficiency and robustness of exogenously determined alternative strategies.

Reports on formal inclusion of future uncertainties in long-term energy-environment modelling are scant. A similar problem has also been addressed with multi-stage stochastic programming [20,21].

3.1 The Two-Stage Recourse Problem

Our formulation is based on the two-stage technique proposed by Dantzig [22], and later extended by Wets [23], as the recourse problem. Consider the following general linear program:

Minimize cx

Subject to

$Ax \geq b$

$x \geq 0$

The two-stage recourse problem formulation allows the elements of $\boldsymbol{c}$, $\boldsymbol{A}$ and $\boldsymbol{b}$ to be random numbers with discreet distributions. Each combination of different $\boldsymbol{c}$, $\boldsymbol{A}$ and $\boldsymbol{b}$ is defined as a

realization. In each realization, there is a choice between satisfying the constraints with the regular decisions, $\boldsymbol{x}$, or through *recourse actions*. Of course the recourse actions have associated penalty costs, the expected value of which is minimized along with the expected value of $\boldsymbol{cx}$. Detailed mathematical description of the recourse problem formulation, as applied in the current problem, is given in Appendix A.

In general the recourse penalties are non-linear convex† cost functions which appear in the objective function. These non-linear cost functions have been constructed as 3-step linear functions in implementing this formulation.

The Uncertainties

Two primary (uncertain) factors affecting long-term energy-environment policy are the end-use energy demand, and the carbon mitigation effort. These uncertainties have been represented by those in macroeconomic growth, and tax on carbon emission, respectively. Three levels of each have been considered.

Growth Scenarios: The High Growth Scenario assumes a compound annual rate of growth (CARG) of 5% for the period 1995-2035. It starts at 6.25% in 1995 and saturates at 2.1% by the year 2100. Medium scenario assumes a CARG of 4.5%, starting at 5.5% and saturating at 2%. Low Scenario CARG is 4%, starting at 4.75% and saturating at 1.9%. The above assumptions are based on the observed GDP trends (**Table 1**) and projections of some other studies [24,25].

Table 1 GDP Trends

Period	1960-70	1970-80	1980-90	1950-90
CARG (%)	3.48	3.58	5.03	4.02

Source: Basic Statistics, CMIE, 1994.

Carbon Tax Scenarios: These are based on the proposals being considered by the IPCC [26,27,28], which is the apex body exploring a global policy to deal with the issue of climate change. The highest tax trajectory follows from the global efforts to stabilize concentration of carbon in the atmosphere at the 1990 level. The global carbon tax trajectory corresponding to stabilization is assumed as derived by a global macroeconomic model analysis [29]. The Reference tax trajectory has been put at 25% level of the stabilization tax in all time periods.

Carbon Tax Scenarios

1. Stabilization Tax (probability = 0.25)
2. Reference Tax (probability = 0.50)
3. Zero Tax (probability = 0.25)

Growth Scenarios

1. High Growth (probability = 0.25)
2. Reference Growth (probability = 0.50)
3. Low Growth (probability = 0.25)

The probabilities have been assigned so as to reflect a *higher* confidence in the reference scenario, than the extreme ones. As there is no firm basis of estimating these probabilities, we have included a sensitivity analysis, which is discussed in section 3.4.

† The recourse actions are ad-hoc measures, which will come at an increasing cost.

Carbon tax and macroeconomic growth have been assumed to be independent random variables. **Table 1** shows the different combinations of carbon tax and growth levels, that form a set of nine possible realisations.

Table 2 Scenarios of Stochastic Indian MARKAL

No.	REALIZATION	PROBABILITY
1	Stabilization Tax with High Growth	0.0625
2	Reference Tax with High Growth	0.1250
3	Zero Tax with High Growth	0.0625
4	Stabilization Tax with Reference Growth	0.1250
5	Reference Tax with Reference Growth	0.2500
6	Zero Tax with Reference Growth	0.1250
7	Stabilization Tax with Low Growth	0.0625
8	Reference Tax with Low Growth	0.1250
9	Zero Tax with Low Growth	0.0625

The recourse actions

The three recourse levels represent different classes of recourse actions. Consider electricity, for example. The first level recourse would be inviting private parties to invest in electricity generation plants. The costs would increase due to the returns given back to the investors. Additionally, the project implementation cost will be higher because of *crashing*. The second level recourse would be purchasing electricity from other sources, located within India or outside. Royalty payments, profits, and transmission infrastructure would increase the costs. The third level recourse represents the economic cost of lost demand.

Penalty costs associated with various recourse actions and recourse fuels have been derived from the dual values of demand constraints and fuel balance constraints respectively. Standard IM solutions under different tax scenarios have been used to determine the recourse penalties for the three carbon tax scenarios under consideration. In the reference stochastic case, the three steps of recourse actions for each end-use demand have been assumed to be twenty five, fifty and seventy five percent higher than its shadow price in the standard IM run. In each period, the first level recourse can meet a maximum of twenty five percent of the additional demand in high growth scenario (over the reference case), and the second level recourse can meet up to fifty percent of the same. The third level recourse actions are unbounded.

3.2 Model Formulation and Solution

The detailed formulation of Stochastic IM (formulated as a two-stage recourse problem) is given in Appendix B. A good description of the standard MARKAL model (a linear program with deterministic parameters) can be found in [30]. The conversion of the standard MARKAL to its stochastic form is outlined below:

Objective Function: Two modifications are made in the objective function: First, the term accounting for the carbon tax is replaced by the expected cost of carbon tax; and second, the expected cost of recourse action penalties is added to the objective function. The penalty costs are broken into non-energy and fuel costs. Further, both components are represented by step-wise linear functions.

Stochastic Demand Constraints: The set of (deterministic) demand constraints is replicated as many times as the number scenarios. Variables representing positive and negative recourse actions are added to each constraint, in each scenario. The negative recourse actions refund the variable costs in realizations which have surplus capacity. Because they refund only the variable cost, the benefit is always less than the cost associated with the corresponding positive recourse. This eliminates the possibility of both negative and positive recourse actions assuming non-zero values in the same constraint.

The above variables and equations have been added to the GAMS code of standard MARKAL. To input the parameters of stochastic IM, extensive modifications have been made in the utility which generates GAMS definitions. With nine probabilistic scenarios, the problem size is 16,027 variables and 6,148 constraints. The computational time increases to about an hour and thirty minutes from under thirty minutes for the standard IM (using a 486/66MHz based PC).

3.3 Results

Results of the stochastic IM show that even in the initial periods, it is optimal to build excess energy supply capacity in anticipation of a higher macroeconomic growth. Further, proportions of coal and natural gas in the aggregate energy supply decrease and increase respectively. This clearly indicates that the probable occurrence of high levels of carbon tax in later periods has a distinct influence on the composition of energy supply in the immediate future.

Results of the stochastic IM have been compared with those of the stabilization tax and high growth scenarios of the standard IM. These scenarios have been chosen because, for increases in aggregate energy supply levels (*growth effects*), the high growth scenario of the standard IM sets the upper bound for stochastic IM results. Similarly, stabilization tax scenario is the upper bound for changes in composition of energy supply (*tax effects*).

Table 3 shows the aggregate energy supply capacity under three different scenarios up to the year 2010. The figures are expressed in terms of their difference over the corresponding values in the reference scenario of standard IM. A similar comparison of installed electricity generation capacity is shown in **Table 4**. The aggregate energy supply capacity closely follows the high growth trajectory in the initial periods. The proportion of natural gas increases and those of coal and oil decrease to confirm the tax effect. A similar trend is observed in the electricity generation sector. There is an increase in aggregate generation capacity with an increase in the proportion of gas based technologies.

Table 3 Difference in Aggregate Energy Supply (Over the Reference Scenario)

SCENARIO	2000	2005	2010	SCENARIO	2000	2005	2010
	COAL				*NATURAL GAS*		
HIGH GROWTH	610.37	1352.09	2000.85	**HIGH GROWTH**	89.64	92.01	144.70
STAB. TAX	-253.22	-1108.78	-2104.65	**STAB. TAX**	48.30	588.31	1279.57
STOCHASTIC	333.72	134.69	76.32	**STOCHASTIC**	41.43	407.79	636.46
	OIL				*TOTAL*		
HIGH GROWTH	270.47	512.90	759.35	**HIGH GROWTH**	999.38	1957.00	2963.27
STAB. TAX	-0.53	-9.98	-31.43	**STAB. TAX**	-176.55	-402.26	-721.17
STOCHASTIC	171.43	295.49	419.36	**STOCHASTIC**	940.37	1701.80	2623.78

Petajoules

Table 4 Difference in Electricity Generation Capacity (Over the Reference Scenario)

SCENARIO	2000	2005	2010	SCENARIO	2000	2005	2010
COAL BASED				*NATURAL GAS BASED*			
HIGH GROWTH	7.54	14.42	21.73	**HIGH GROWTH**	2.6	4.25	6.65
STAB. TAX	-0.36	-11.85	-19.02	**STAB. TAX**	0.13	9.64	16.43
STOCHASTIC	2.66	-2.58	-4.52	**STOCHASTIC**	0.77	8.68	12.74
				TOTAL			
				HIGH GROWTH	10.48	18.67	29.51
				STAB. TAX	0.11	0.19	-0.85
				STOCHASTIC	3.77	6.1	9.35

Gigawatts

3.4 Sensitivity Analysis

The results discussed in the previous section hold for the assumed probabilities of various realizations. It is useful to study the behaviour of this solution under different levels of uncertainty because it is difficult to have its apriori estimate. This section describes the sensitivity analysis on the scenario probabilities.

Results of the reference case for stochastic MARKAL (SM-REF) have been compared with those of a high uncertainty scenario (SM-HI-UN) and a low uncertainty scenario (SM-LO-UN). These scenarios express ten percent lower and higher confidence in the reference scenario, respectively. Symmetrical distributions have been used for a valid comparison with the reference scenario. The probability distributions corresponding to these scenarios are given in **Table 5**.

Table 5 Stochastic IM Scenarios for Sensitivity Analysis

SCENARIO PARAMETERS	SM-REF	SM-LO-UN	SM-HI-UN
High Tax Probability	0.25	0.15	0.30
Reference Tax Probability	0.50	0.70	0.40
Zero Tax Probability	0.25	0.15	0.30
High Growth Probability	0.25	0.15	0.30
Reference Growth Probability	0.50	0.70	0.40
Low Growth Probability	0.25	0.15	0.30

Results show that the growth effect is highly sensitive to the level of uncertainty. But the tax effect shows only a small variation with a change in the level of uncertainty.

Figure 1 shows the aggregate energy supply capacity under various scenarios. The difference between capacities in the extreme scenarios and the respective bounds is less than 2.5%. Thus, the growth effect has traversed its entire range in the narrow range of probabilities. The relatively lower sensitivity of tax effect is evident from **Figure 2** and **Figure 3**, which show the annual carbon emission and coal based generation capacity, in various scenarios. Clearly, the coal based capacity does not show any significant variation across the different uncertainty scenarios. The gas based generation capacity shows a wide variation to accommodate the growth effect, as shown in **Figure 4**.

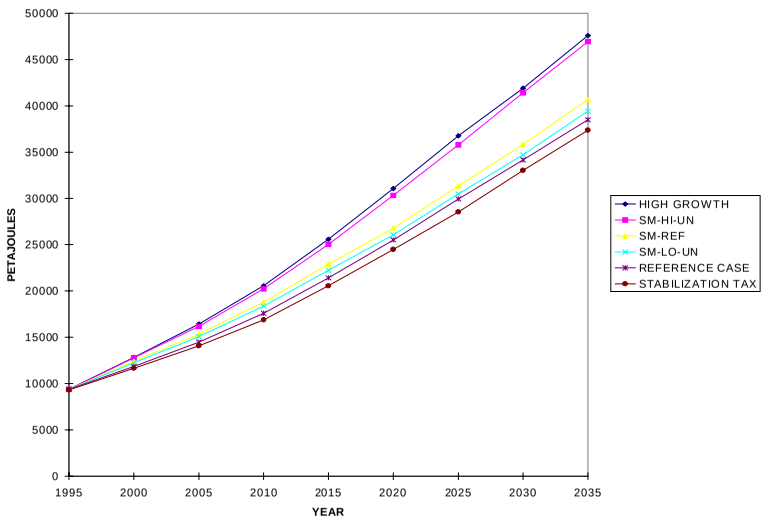

Figure 1 AGGREGATE ENERGY SUPPLY CAPACITY

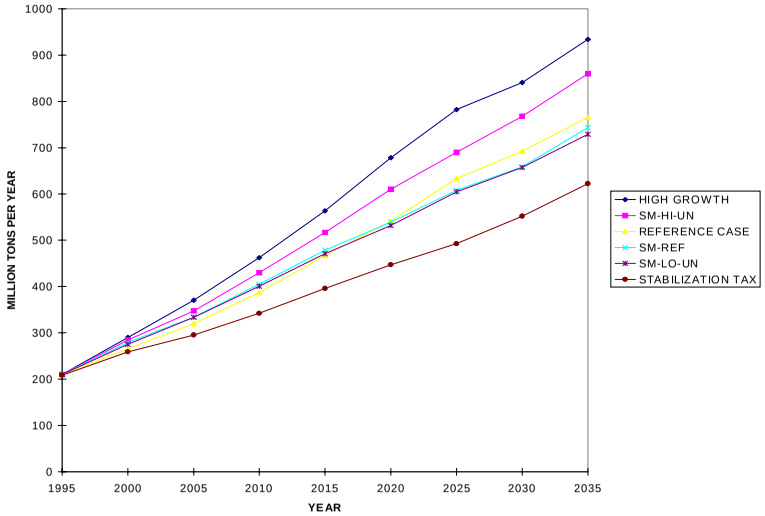

Figure 2 ANNUAL CARBON EMISSION

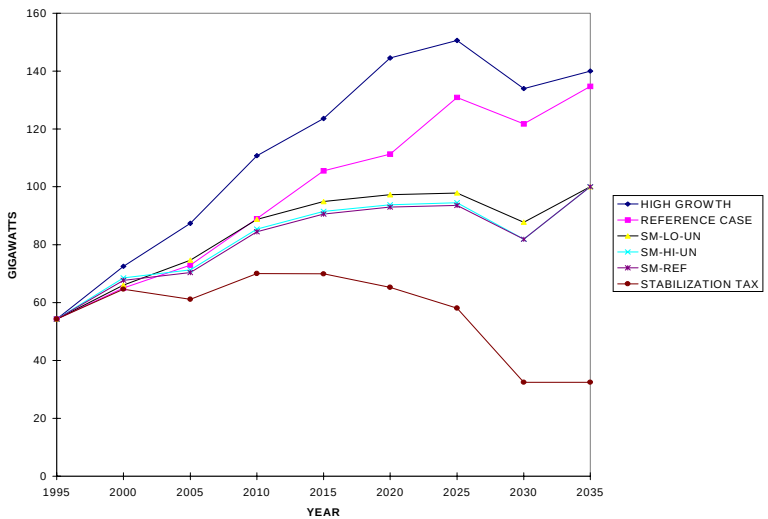

Figure 3 COAL BASED ELECTRICITY GENERATION CAPACITY

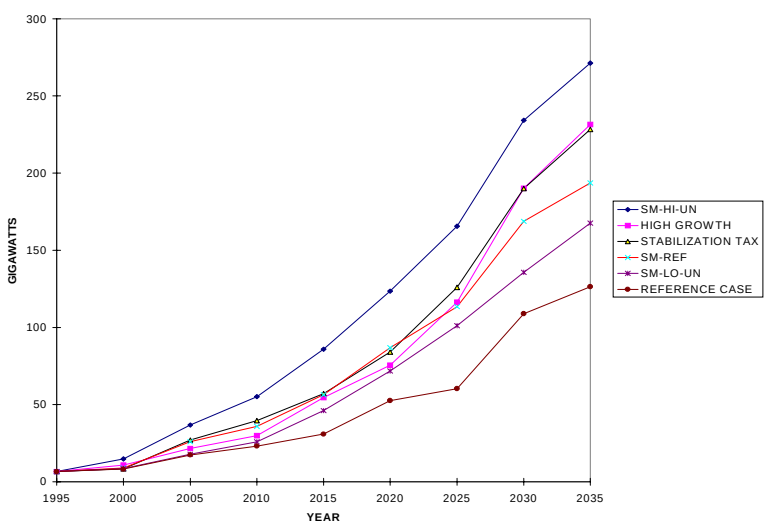

Figure 4 GAS BASED ELECTRICITY GENERATION CAPACITY

4. Elastic Indian MARKAL

The carbon mitigation instruments increase the cost of various energy services. This imposes a significant strain on different end-use sectors of the economy. In this section, a variant of the standard version, the Elastic IM has been used to analyse the impact of carbon mitigation instruments on the energy system with price sensitive end-use demands. The formulation of this model is based on that of the MARKAL-ED [31].

Price elastic demands have been used in bottom-up energy models earlier. For example, price elasticity of demands was considered in the PIES model [32]. But as PIES is an energy supply model, the demand was specified at the fuel level rather than the end-use level, as it has been done in the standard IM and MARAKL-ED. MARKAL-MACRO [33] incorporates the price sensitivity of end-use demands but has a single elasticity coefficient for all end-use sectors. MARKAL-ED can incorporate variations in price elasticities across different end-use sectors.

4.1 Model Formulation and Solution

The deterministic right hand side of the demand constraints has been replaced by a price sensitive demand function, given below:

$$D(p) = Kp^{-e}$$

$$D(p) = D_0\left(\frac{p}{p_0}\right)^{-e}$$

where, ***D(p)*** is the price sensitive end-use demand, ***p*** is the price and ***e*** is the own price elasticity of demand ***D(p)***. (D_0, p_0) is a known point on the demand curve. This is implemented through changes in the objective function and the demand constraint, which are described in Appendix C.

Only a price increase is considered because the primary purpose of this formulation is to examine the impact of carbon mitigation measures, which increase energy prices. The maximum possible demand reduction has been assumed to be twenty percent of the reference scenario demand level. This has been derived from a macroeconomic study of carbon mitigation scenarios for India [34], which predicts a GDP loss of up to fourteen percent under the stabilization tax scenario. This translates into a twenty percent decrease in energy demand with the GDP elasticity of energy demand equal to 1.4.

The demand curve has been incorporated by adding a *dummy supply* in the end-use demand constraints. The dummy supply does not consume any energy resources, and can supply a maximum of twenty percent of the reference scenario demand.

The non-linear dummy supply function has been approximated by a 5-step linear function using 5 different dummy technologies. Each technology is offered with an upper bound of $D_0/25$, where $\boldsymbol{D_0}$ is the reference case demand. Each technology incurs an annual cost per unit of demand met, which is evaluated based on the demand equation and the shadow price of the demand constraint under reference scenario.

With the above transformation of the dummy supply function, the problem can be solved within the standard IM framework. The problem size is 5,317 variables and 3,628 constraints for 5 dummy technologies each in 7 end-use demand categories. There is no significant change in computational time over the standard IM.

The price elasticity of demands

Most markets in India are in a state of non-equilibrium because of a constrained supply and administered prices. Thus, demand is greatly influenced by the supply. Over the years, both supply and demand have increased along with the prices. Due to this reason, an econometric analysis often yields negative price elasticities of demand. These elasticities will gradually assume positive values with the process of economic development. For examining the impact of carbon mitigation instruments, it has been assumed that the elasticities are close to zero in the year 1995 and saturate at 0.5 by 2050, following an 'S' curve pattern. The same trend of elasticities has been assumed for all seven end-uses under consideration. The end-uses are given in **Figure 5**.

4.2 Results

The different end-use demands reduce from 4 to 20 percent in response to carbon tax. Percentage reduction in demand is shown in **Figure 5**. There is a wide variation in reduction despite the fact that each end-use has been assumed to have the same price elasticity. This has happened because of the difference in availability of low carbon technological options to the various demand categories. There is an increase in the percentage reduction across time periods because (a) the price elasticity increases in later periods, and (b) the energy prices are higher due to higher carbon tax levels in the later periods. Demand reduction contributes up to a maximum of ten percent to the overall carbon emission reduction under the stabilization tax scenario.

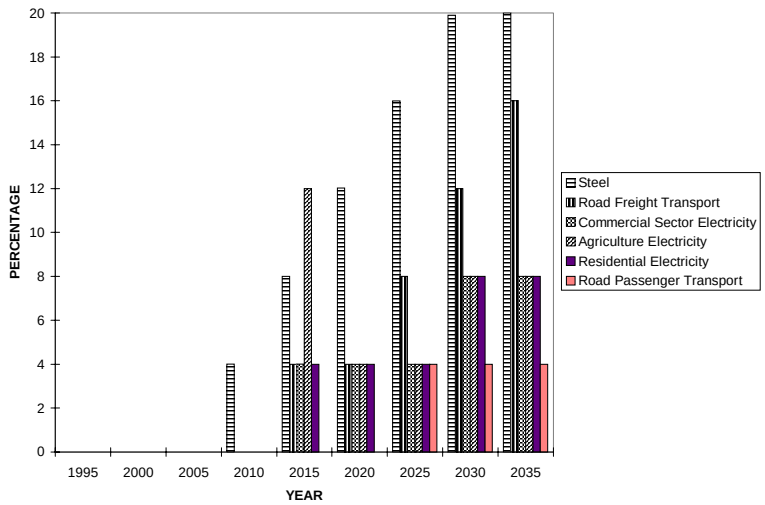

Figure 5 PERCENTAGE LOSS IN END-USE DEMANDS

5. Conclusion

In this paper, we have shown a new way to accommodate future uncertainties in energy-environment planning using the two-stage recourse problem formulation. The formulation

has been reduced to a linear form so that it is possible to handle even large models easily. This approach is particularly suitable for developing countries, where planning for shortages is an every day reality. The recourse actions make it possible to produce a base plan which would actually fall short of the requirements under some scenarios. This cannot be done with the multi-stage stochastic programming approach [20, 21] for example, where the options available within the model have to deliver a feasible solution under all scenarios. Including uncertainties in his manner can make the results of such long term technology evaluation models more relevant for the policy makers.

The elastic IM confirms that a part of the macroeconomic impact of carbon taxes can be easily captured in a detailed bottom-up analysis, within a linear formulation, as first suggested by Loulou [25]. Further, it gives a useful estimate of emission reduction due to demand loss under severe carbon taxes.

Appendix A: The Two-Stage Recourse Problem

Consider the following general stochastic linear program:

$$Minimize\ z = c^T x$$

Eq. 1

where, ***c*** = (c_1, c_2, .. c_n)^T, and ***x*** = (x_1, x_2, .. x_n) are the decision variables,

$$Subject\ \ to:$$
$$Ax = b$$
$$Tx = g$$
$$x \geq 0$$

Eq. 2

A = (a_{ij}) is an *m* x *n* matrix, ***b*** = (b_1, b_2, .. b_m)^T, ***T*** is a deterministic *r* x *n* matrix and ***g*** is a deterministic column vector. Elements of vectors ***b***, ***c*** and the rows of ***A*** are random variables with known probability distributions. The fixed constraints ***Tx = g*** are assumed to have a feasible solution.

Let

$$y_i = b_i - \sum_{j=1}^{n} a_{ij} x_j$$
$$\forall i = 1,..,m$$

Eq. 3

Let f_i be the penalty associated with the discrepancy y_i. Then, the second stage of the formulation is given by (4),

$$Minimise \sum_{i=1}^{m} f_i |y_i|$$

Eq. 4

subject to (3), where ***b***, ***x***, ***A*** and ***f***=(f_1,..,f_m)^T are known. The final two-stage program is given as (5).

$$Minimise \quad E[c^T x + \min_{y^+, y^-} f^T (y^+ + y^-)]$$
$$s.t.$$
$$Iy^+ - Iy^- = b - Ax$$
$$Tx = g$$
$$x, y^+, y^- \geq 0$$

Eq. 5

where, ***E*** denotes the expected value.

In the context of the problem at hand, the penalty costs associated with the discrepancies (recourses) comprise energy and non-energy components. The energy component is specific to the fuel used by a recourse whereas the non-energy component is recourse specific. The total recourse fuel usage is given in (6),

$$\sum_{i=1}^{m} RFC_{ji} |y_i| = RFU_j$$
$$\forall j = 1,..,s$$

Eq. 6

where, ***RFC*** is an (*m*x*s*) matrix with RFC_{ij} is the consumption coefficient of fuel ***j*** for recourse ***i***. RFU_j=(RFU_1,...,RFU_s) is the total supply of recourse

fuels. Thus, the total penalty cost associated with the recourses is given by (7),

$$E[\sum_{i=1}^{m} f_i |y_i| + \sum_{i=1}^{m} \sum_{j=1}^{s} RFC_{ij} |y_i| FC_j$$

Eq. 7

where FC_j is the unit cost of fuel ***j***.

Appendix B: Model Formulation

Let the end-use demands (***demand$_k$(t)***) and carbon tax (***ctax(t)***) be random variables with discreet probability distributions. Further, let the joint probability distribution be characterized by a random vector ***z*** given in equation (8).

$$\zeta^l = (ctax^l(t), demand_k^l(t))$$

$$p^l \geq 0$$

$$\sum_{l=1}^{L} p^l = 1$$

$$\forall l = 1,..,L, k \in DM, t \in T$$

Eq. 8

Sets

DM	=	Class of all end-use demands
DMD	=	Class of all end-use demand technologies (a subset of class TCH)
DMD	=	Subset of the end-use technologies that can satisfy end-use demand ***k*** ∈ DM
ELA	=	Class of all electricity producing technologies
ENC	=	Class of all energy carriers, excluding electricity
GRD	=	Set of grades of technologies/energy sources
PRC	=	Class of process technologies
T	=	Set of time periods
TCH	=	Set of all technologies
u=(1,...,U)	=	Price levels
W	=	Set of extraction and renewable sources
Y	=	Set of daily divisions (***d*** refers to day division, ***n*** to night division)

Parameters

a	=	Annual discount rate
$af_i(t)$	=	Availability factor of technology ***i*** in time period ***t***
$cf_i(t)$	=	Average utilization factor of installed capacity of demand device ***i*** in period ***t***
$cost_{ks}(t)$	=	Unit cost of energy carrier ***k*** from source ***s*** at time period ***t***
$crbcoeff_i(t)$	=	Carbon emission per unit activity of energy supply technologies and per unit capacity of demand technologies in time period ***t***
ctax(t)		Carbon tax in time period ***t***
$demand_k(t)$	=	Useful Energy demand of demand category ***k***, in time period ***t***
$epk_i(t)$	=	Fraction of the consumption of electricity by process technology ***i*** which is to be included in the peaking relation at period ***t***
er(t)	=	Required capacity reserve for the electricity grid in time period ***t***
$fixom_i(t)$	=	Annual fixed operating and maintenance cost associated with the installed capacity of technology ***i*** in time period ***t***
fo_i	=	Forced outage of electricity generation technology ***i***
$fr_{j(i)y}$	=	Fraction of demand from demand category ***J(i)*** which occurs during time of day ***y***
$inp_{ki}(t)$	=	Input of energy carrier ***k*** per unit of activity of technology ***i*** (for non-demand technologies)
$invcost_i(t)$	=	Unit investment cost of technology ***i*** at time period ***t***
j(i)	=	Function indicating the demand sector ***j***(***i***)∈DM to which demand technology ***i*** belongs
l_i	=	Lifetime of technology ***i***
n	=	Number of years in each time period
η(t)	=	Transmission and distribution efficiency of the grid in time period ***t***
$out_{ki}(t)$	=	Output of energy carrier ***k*** per unit of activity of technology ***i*** (for non demand technologies) in period ***t***
$pk_i(t)$	=	Fraction of installed capacity of generating plant ***i*** available to meet peak
$price_{ks}(t)$	=	Unit price of energy carrier ***k*** to source ***s*** at time period ***t***
q_y	=	Fraction of the year occurring in time of day ***y***
$resid_i(t)$	=	Capacity installed before start of optimization available for use in period ***t*** and for which no investment cost is charged
$rfbound_j^u(t)$	=	Upper bound on the supply of resource fuel ***j***, at price level ***u***, in time period ***t***
$rfc_{jk}^r(t)$	=	Amount of fuel ***j***, used for unit activity of recourse ***k***, at level ***r***, in time period ***t***
$rfcost_j^{lu}(t)$	=	Unit price of recourse fuel ***j***, at price level ***u***, in period ***t*** with $ctax^l(t)$
t_o	=	Max {1, ***t***-l_i + 1}
u_i	=	Factor converting units of capacity to units of annual production
$varom_i(t)$	=	Annual variable operating maintenance cost of the non-demand technology ***i*** in time period ***t***
$ybound_k^r(t)$	=	Upper bound for recourse ***k***, at level ***r***, in time period ***t***

Decision Variables

$CAP_{tch,t}$	=	Capacity for technology TCH during time period ***t***
$C_i(t)$	=	Capacity of technology ***i*** in period ***t***
CRBEMM(t)	=	Total carbon emission from the energy system in time period ***t***
$EXP_{ks}(t)$	=	Export of energy carrier ***k*** to destination ***s*** in period ***t***
$I_i(t)$	=	New capacity of technology ***i*** installed at beginning of period ***t***
$IMP_{ks}(t)$	=	Import of energy carrier ***k*** from source ***s*** in period ***t***
$MIN_{ks}(t)$	=	Supply of primary energy form ***k*** from extraction or renewable source ***s*** in time period ***t***
$P_i(t)$	=	Activity of technology ***i*** in period ***t***
$P_{iy}(t)$	=	Electricity production by technology ***i*** in period ***t*** for time of day ***y***
$RFSUP_j^{lu}(t)$	=	Amount of recourse fuel ***j*** supplied at price level ***u***, in time period ***t*** and realization ***l***
$Y_k^{lr}(t)$	=	Recourse action for demand ***k***, at level ***r***, in time period ***t***
$YN_k^l(t)$	=	Negative recourse action for demand ***k***, in time period ***t***

Stochastic Indian MARKAL Model:

$$Z = Minimize \sum_{t\in T} \frac{1}{(1+\alpha)^{n(t-1)}} \sum_{i\in TCH} inv\cos t_i(t) * I_i(t)$$
$$+ \sum_{t\in T} \frac{1}{(1+\alpha)^{n(t-1)}} (\sum_{m=1}^{n} (1+\alpha)^{1-m}) * [\sum_{i\in TCH} fixom_{iG}(t) * C_{iG}(t)$$
$$+ \sum_{i\in PRC} \operatorname{var} om_i(t) * P_i(t) + \sum_{i\in ELA} \sum \operatorname{var} om_i(t) * P_{iy}(t)$$
$$+ \sum_{k\in ENC} \sum_{w} \cos t_{kw}(t) * MIN_{kw}(t) + \sum_{k\in ENC} \sum_{s} \cos t_{ks}(t) * IMP_{ks}(t)$$
$$- \sum_{k\in ENC} \sum_{s} price_{ks}(t) * EXP_{ks}(t) + ctax(t) * CRBEMM(t)$$

Eq. 9

$$+ \sum_{t\in T} \frac{1}{(1+\alpha)^{n(t-1)}} (\sum_{m=1}^{n} (1+\alpha)^{1-m}) * [\sum_{l=1}^{L} p^l * \sum_{r\in R} \sum_{k\in DM} f_k^{lr}(t) * Y_k^{lr}(t)]$$

Eq. 10

$$- \sum_{t\in T} \frac{1}{(1+\alpha)^{n(t-1)}} (\sum_{m=1}^{n} (1+\alpha)^{1-m}) * [\sum_{l=1}^{L} p^l * \sum_{r\in R} \sum_{k\in DM} fn_k^{lr}(t) * YN_k^{lr}(t)]$$

Eq. 11

$$+ \sum_{t\in T} \frac{1}{(1+\alpha)^{n(t-1)}} (\sum_{m=1}^{n} (1+\alpha)^{1-m}) * [\sum_{l=1}^{L} p^l \sum_{j=1}^{s} RF_j^l(t) * rf\cos t(RF_l^l t)]$$

Eq. 12

Subject to:

$$C_i(t) = \sum_{m=t_0}^{t} I_i(m) + resid_i(t)$$
$$\forall i \in TCH, t \in T$$

Eq. 13

$$\sum_{\substack{i\in TCH \\ i\notin DMD}} out_{ki}(t) * P_i(t) + \sum_{s} IMP_{ks}(t) + \sum_{w} MIN_{kw}(t) \geq \sum_{\substack{i\in TCH \\ i\notin DMD}} inp_{ki}(t) * P_i(t)$$
$$+ \sum_{i\in DMD} inp_{ki}(t) * cf_i(t) * C_i(t) + \sum_{s} EXP_{ks}(t)$$
$$\forall k \in ENC, w \in W, t \in T$$

Eq. 14

$$n * \sum_{x=1}^{t} MIN_{kw}(x) \leq cumres_{kw}$$
$$\forall k \in ENC, w \in W, t \in T$$

Eq. 15

$$\eta(t) \sum_{i\in ELA} P_{iy}(t) \geq \sum inp_{ELC,i}(t) * cf_i(t) * fr_{j(i)y}(t) * C_i(t) + \sum_{i\in PRC} inp_{ELC,i}(t) * q_y * P_i(t)$$
$$\forall y \in Y, t \in T$$

Eq. 16

$$P_i(t) \leq af_i(t) * C_i(t)$$
$$\forall i \in PRC, t \in T$$

Eq. 17

$$P_{iy}(t) \leq u_i * q_y * af_i(t) * C_i(t)$$
$$\forall y \in Y, i \in ELA, t \in T$$

Eq. 18

$$\frac{\eta(t)}{1+er(t)} \sum_{i\in ELA} u_i * pk_i(t) * C_i(t) \geq \sum_{i\in DMD} inp_{ELC,i}(t) * cf_i(t) * \frac{fr_{j(i)d}}{q_d} * C_i(t)$$
$$+ \sum_{i\in PRC} inp_{ELC,i}(t) * epk_i(t) * P_i(t)$$
$$\forall t \in T$$

Eq. 19

$$CRBEMM(t) = \sum_{y\in Y} \sum_{i\in ELA} crbcoeff_i(t) * P_{iy}(t) + \sum_{i\in PRC} crbcoeff_i(t) * P_i(t)$$
$$+ \sum_{i\in DMD} crbcoeff_i(t) * C_i(t)$$
$$\forall t \in T$$

Eq. 20

$$\sum_{i\in \underline{DMD}} C_i(t) + \sum_{r\in R} Y_k^{lr}(t) - YN_k^l(t) \geq demand_k^l(t)$$
$$\forall l = 1,..,L, k \in DM, t \in T$$

Eq. 21

$$Y_k^{lr}(t) \leq ybound_k^r(t)$$
$$\forall l = 1,..,L, r \in R, k \in DM, t \in T$$

Eq. 22

$$Y_l^{lr}(t) * YN_k^l(t) = 0$$
$$\forall l = 1,..,L, r \in R, k \in DM, t \in T$$

Eq. 23

$$\sum_{k\in DM} \sum_{r\in R} rfc_{jk}^r(t) * Y_k^{lr}(t) \leq \sum_{u=1}^{U} RF_j^l(t)$$
$$\forall l = 1,..,L, j = 1,..,s, t \in T$$

Eq. 24

Some parts of the objective function and equations are briefly described below.

Objective Function(9-12): The first term added to the objective function (10) represents the discounted sum of expected value of recourse penalties. $f_k^{lr}(t)$ is the non-energy cost associated with unit activity of recourse $Y_k^{lr}(t)$. The second expression (11) represents the benefit given to the system in case of a surplus capacity. This is to account for the excess consumption in case of a lower demand realization. $fn_k^{lr}(t)$ is the fuel cost associated with unit activity of negative recourse $YN_k^{lr}(t)$. Expression (12) represents the expected cost of recourse fuel supply, which is a function of the volume of supply.

Stochastic Demand Constraint (21): One constraint for each end-use demand in each realization ensures that the sum of reference case capacities and recourse actions, are at least equal to the demand in each realization. $Y_k^{lr}(t)$ is the recourse action for demand ***k***, level ***r***, in realization ***l***, in time period ***t***. $YN_k^l(t)$ is the negative recourse for demand ***k***, in realization ***l***, for time period ***t***. ***R*** is the set of possible levels of recourse actions.

Recourse Action Bounds (22): The recourse costs have been considered at three different levels. Equation (22) puts upper bounds on individual recourse levels. $\boldsymbol{ybound_k^r(t)}$ is the upper bound for recourse ***k***, at level ***r***, in time period ***t***.

Equation (23), ensures that in any realization, only one of recourse actions (positive or negative) assume a non-zero value.

Recourse Fuel Balance Constraints (Equation 24): These constraints aggregate each energy form across all recourse actions in each realization. $\boldsymbol{rfc_{jk}^r(t)}$ is the amount of fuel ***j***, used for unit activity of recourse ***k***, at level ***r***, in time period ***t***. $\boldsymbol{RF_j^l}$ is the aggregate consumption of recourse fuel ***j*** in realization ***l***.
The non-linear recourse fuel costs have been approximated by a three step linear function, as shown in Equations (25) to (27).

$$RF_j^l(t) \le \sum_{u=1}^{U} RFSUP_j^{lu}(t)$$
$$\forall j = 1,..,s, l = 1,..,L, t \in T$$

Eq. 25

$$RFSUP_j^{lu}(t) \le rfbound_j^u(t)$$
$$\forall j = 1,..,s, l = 1,..,L, t \in T$$

Eq. 26

$$\sum_{t\in T} \frac{1}{(1+\alpha)^{n(t-1)}} (\sum_{m=1}^{n} (1+\alpha)^{1-m} [\sum_{l=1}^{L} p^l \sum_{j=1}^{s} \sum_{u=1}^{U} RFSUP_j^{ul}(t) * rf\cos t_j^u(t)]$$

Eq. 27

Equation (25) expresses the aggregate supply of each recourse fuel as the sum of three different grades having deterministic costs $\boldsymbol{rfcost_j^u}$. Index ***u*** represents the price level. Equation (26) puts bounds on individual grades to represent a step function. The expression (12) in the objective function is replaced by (27) above.

Appendix C: Elastic Indian MARKAL Formulation

The demand constraint of Elastic IM is given below:

$$\sum_{i\in \underline{DMD}} C_i(t) + DUMMYSUPPLY(DV_{kt}, t) \ge demand_k(t)$$
$$\forall k \in DM, t \in T$$

where, $\boldsymbol{DV_{kt}}$ is the dual value of the demand constraint for end-use category ***k***, in time period ***t***. The part of the demand met by dummy supply is considered as lost demand, and the total cost of dummy supply represents the economic cost of lost demand.

The non-linear supply function is represented by step-wise linear function, DS_{kt}^s, where the index ***s*** represents the *steps*. Then, the demand constraint is:

$$\sum_{i\in \underline{DMD}} C_i(t) + \sum_{s\in S} DS_k^s(t) \ge demand_k(t)$$
$$\forall k \in DM, t \in T$$

S is the set of steps.

At each step, the cost associated with the dummy supply, p_k^s, can be evaluated from the demand function as the following expression:

$$p_k^s = \frac{p_k^0}{D_k^{-1/e}} \left(D_k - s \times \frac{D_k}{25} + \frac{1}{2}\frac{D_k}{25} \right)^{-1/e}$$

where, D_k is the reference case demand. Thus the total cost of lost demand included in the objective function is:

$$\sum_{k\in DM} \sum_{s\in S} p_k^s DS_k^s(t)$$

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