VTMXers

Analytical modeling of slope flows

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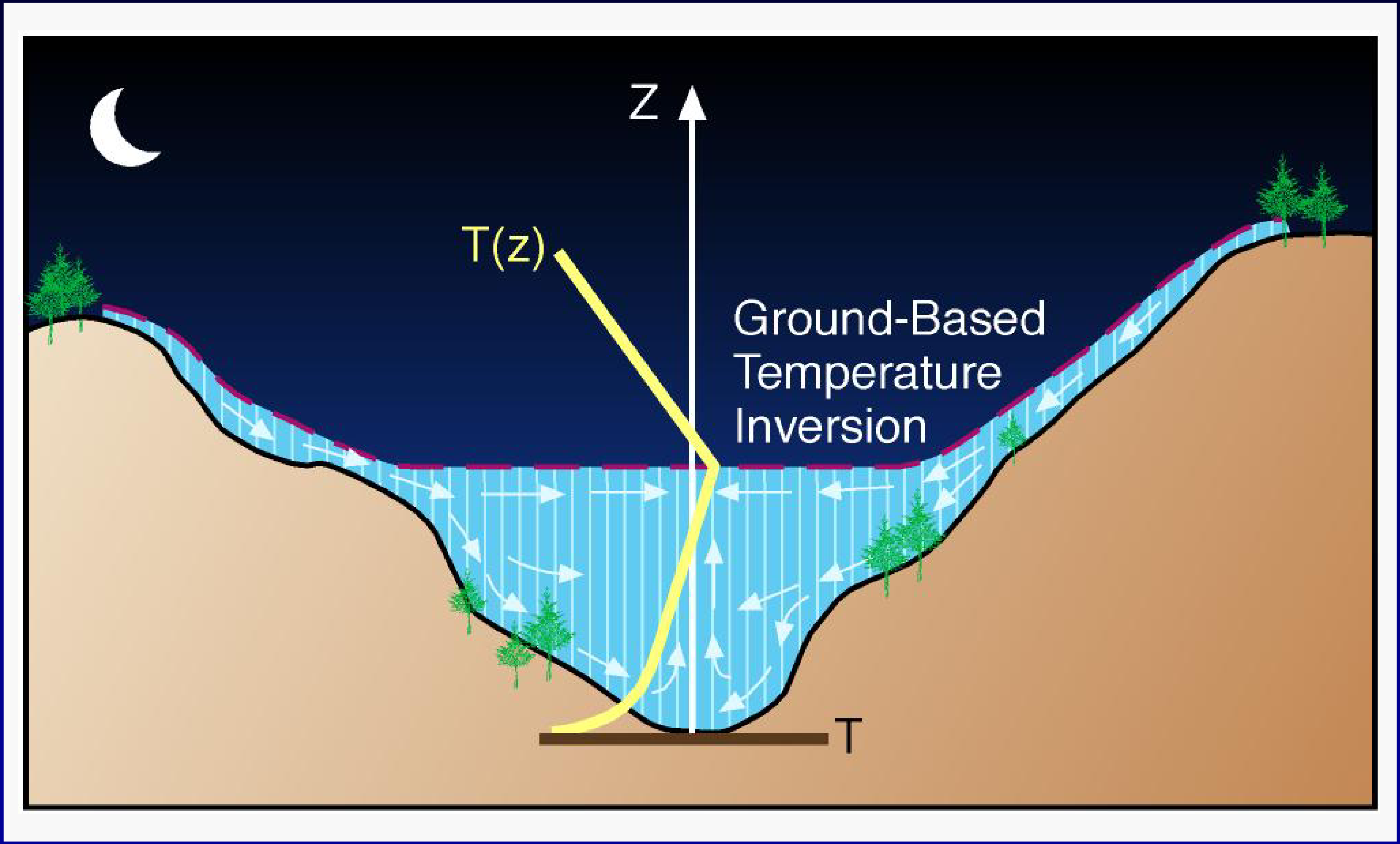
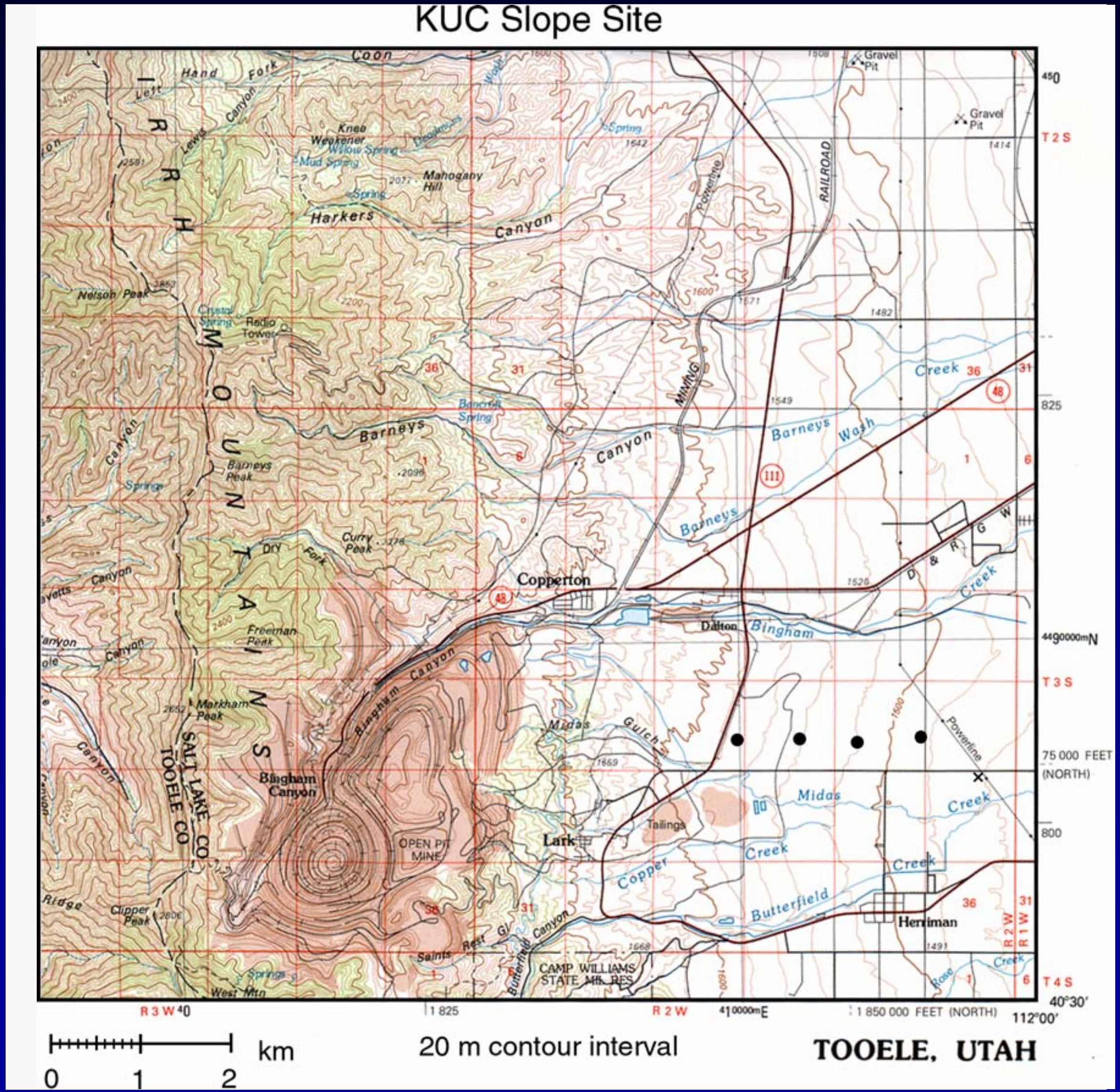
- Momentum and heat budgets
- The local equilibrium concept
- Along-slope flow evolution
- Test of the Prandtl model
- Further work

Downslope flows leave the slope…

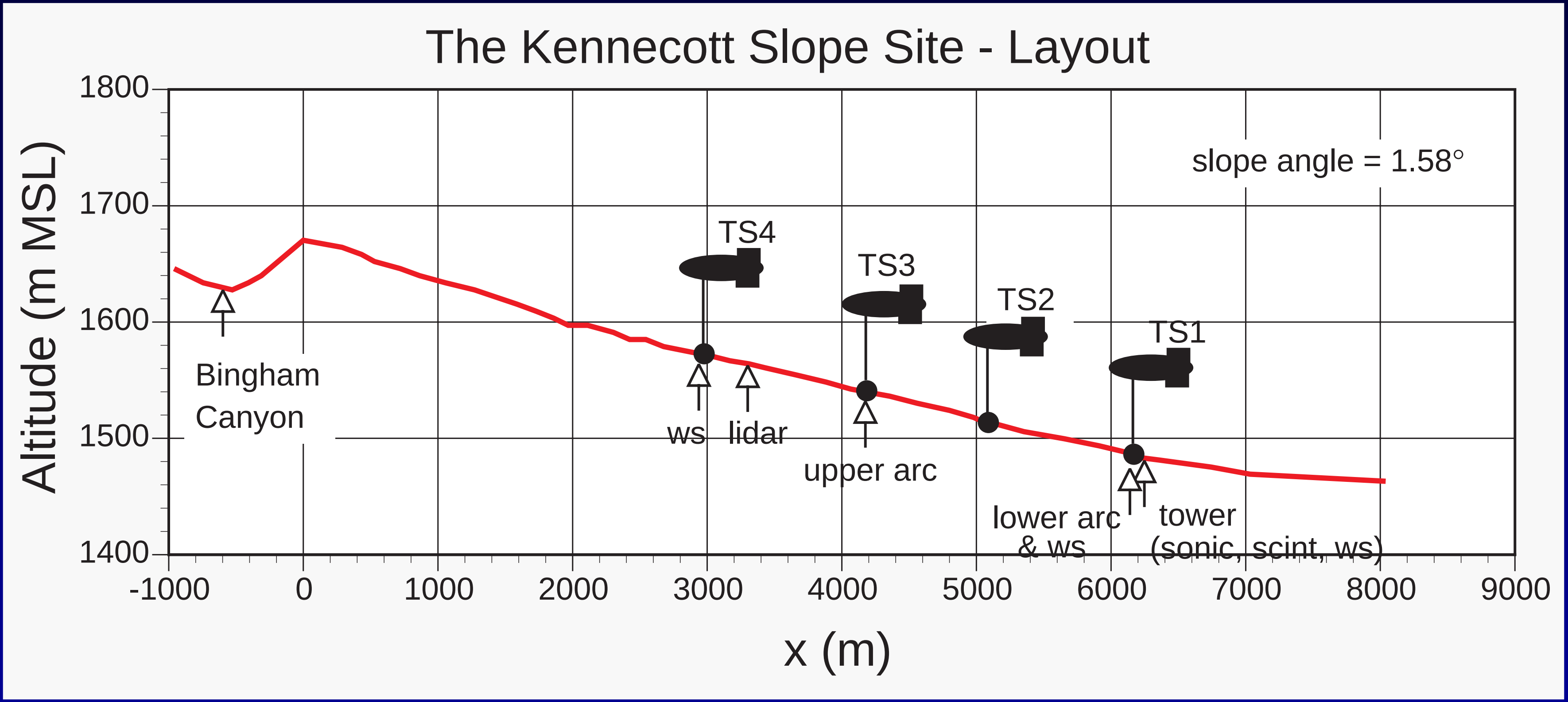


Large-scale topo map

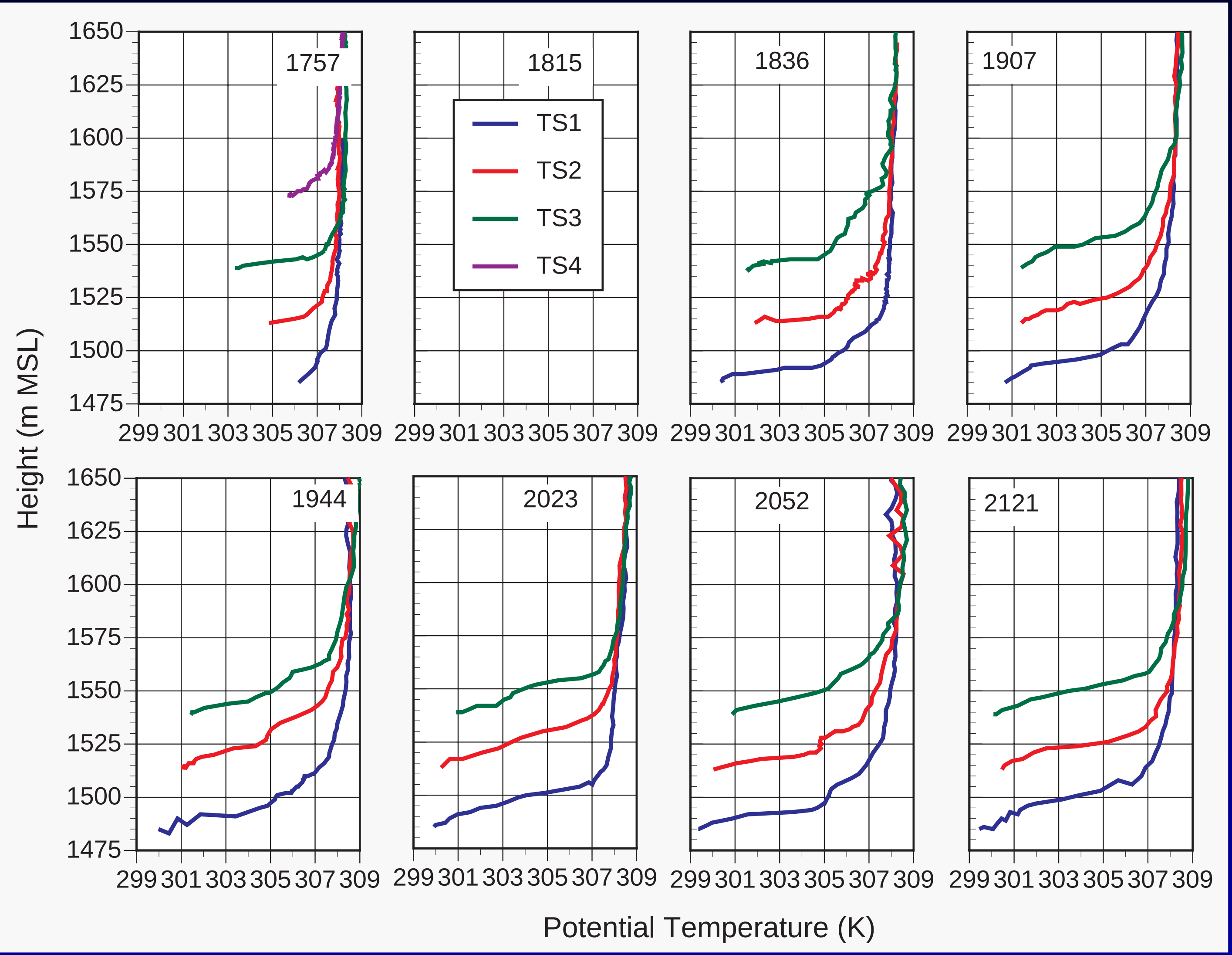


Kennecott Slope, UT

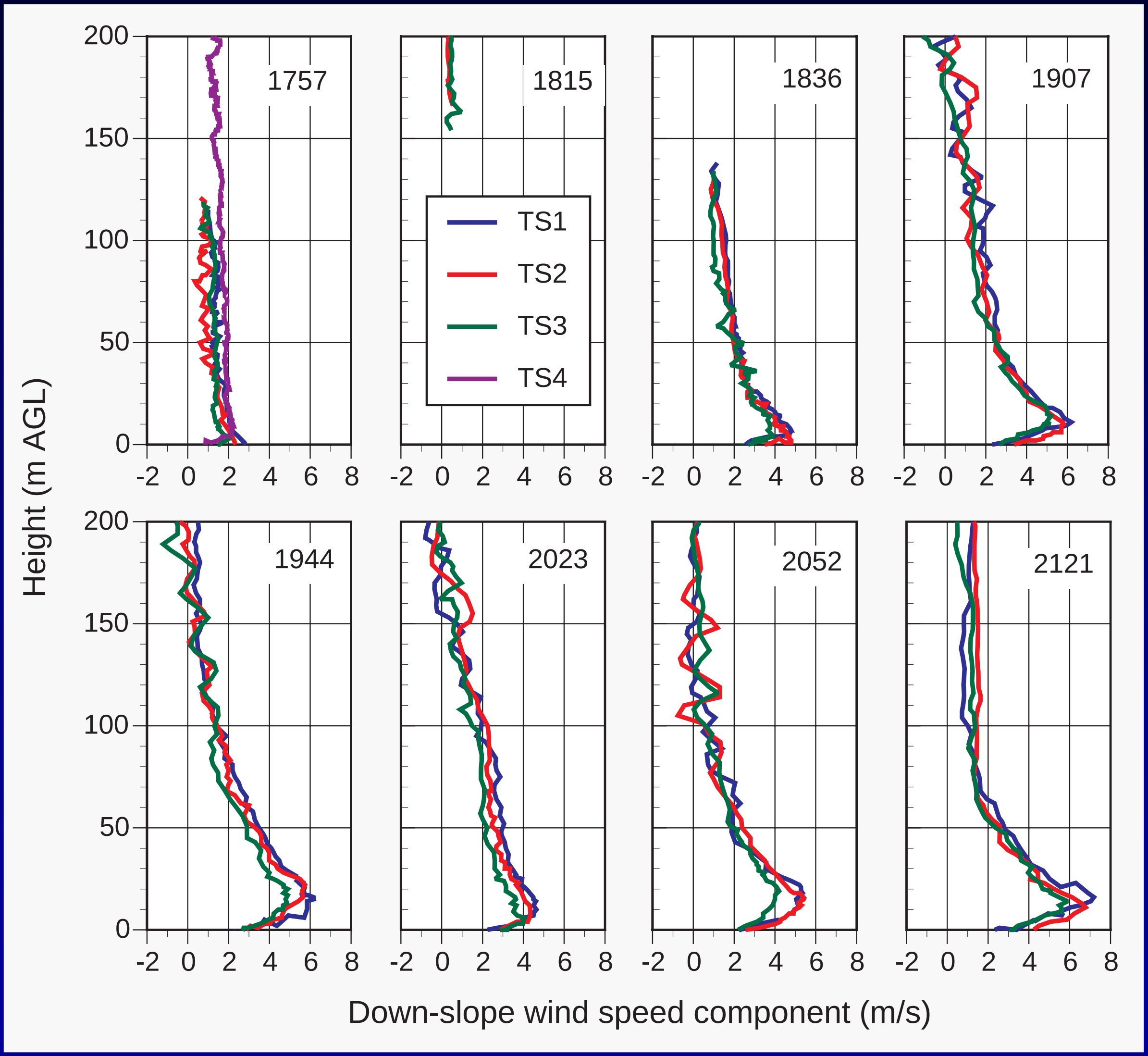
X-section of Kennecott slope



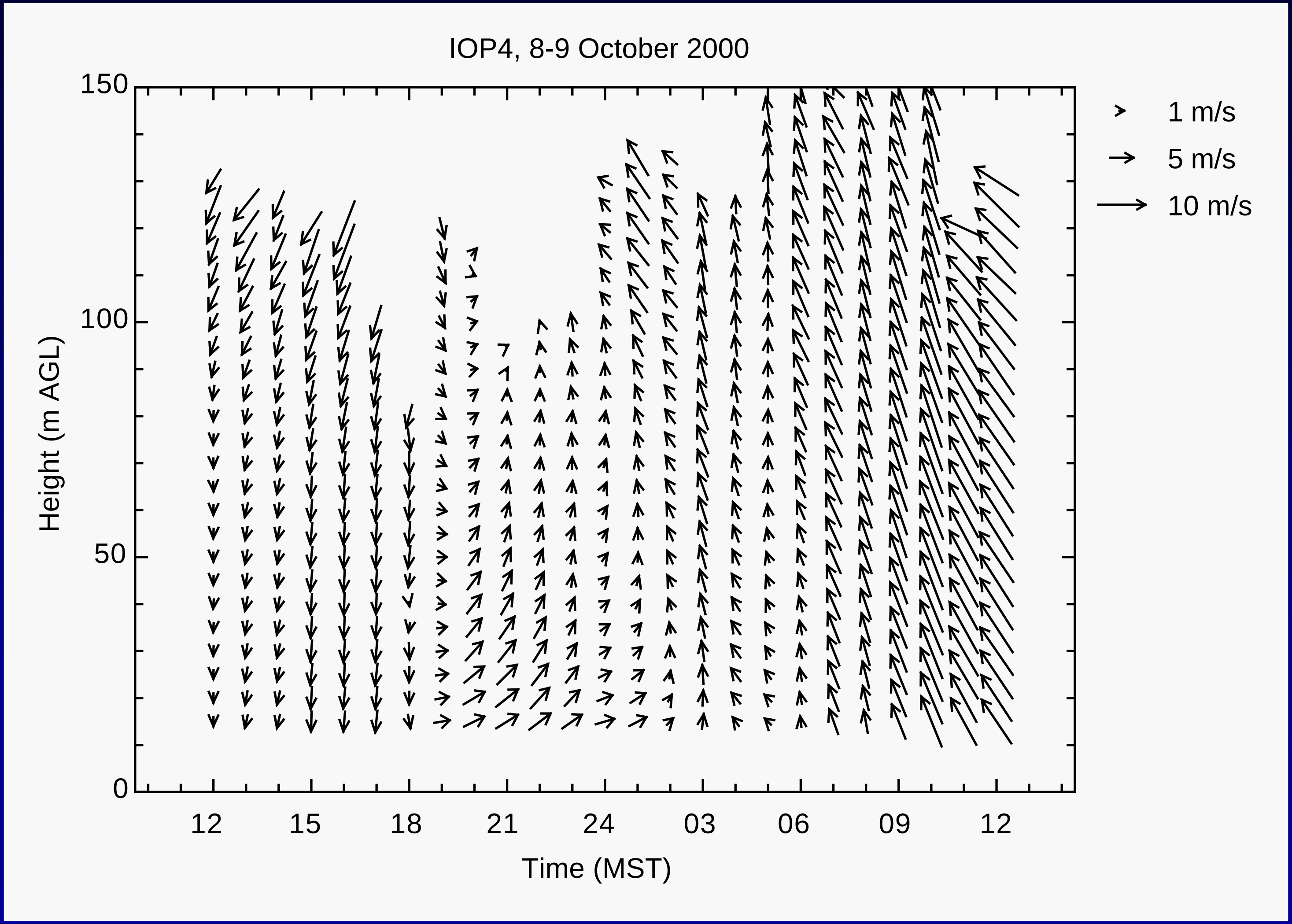
TS1 potential temperature soundings, 2 Oct 2000



TS1-4 downslope wind comp sndgs (AGL), 2 Oct 2000



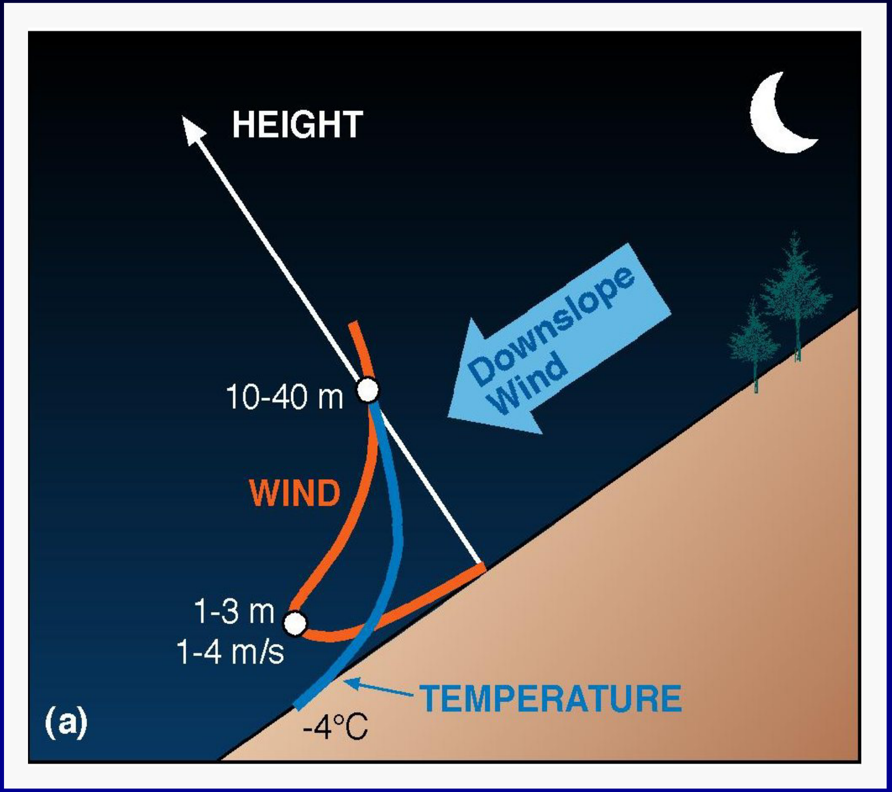
Minisodar winds



data from Rich Coulter, ANL

Why Study Slope Flows in VTMX?

Cold air flows downhill
Air coldest close to ground
Existence of a jet profile
 + Jet max height
 + Jet max speed
 + Inversion strength
 + Bulk fluxes (mass, mom, heat)



External parameters:

- Slope angle
- Roughness length
- Ambient stability
- Thermal forcing (sfc heat flux)

Bulk momentum equation

$$\underbrace{\frac{\partial}{\partial t} h\bar{u}}_{STOR} + \underbrace{\frac{\partial}{\partial x} h\overline{u^2}}_{ADV} = \underbrace{\frac{g}{\theta_0} \sin\alpha\, h\bar{\theta}}_{BUOY} - \underbrace{\frac{g}{\theta_0} \cos\alpha \frac{\partial}{\partial x} h^2 \bar{\bar{\theta}}}_{THERM} + \underbrace{\tau_0 - \tau_h}_{DRAG} + \underbrace{-(w\, u)_h}_{KIN.ENTR}$$

Mahrt (1982)

Assumptions

- Plane slope, no crosswind
- Coriolis effects negligible
- Quasi-hydrostatic balance

Drag parameterization

$$\tau_0 - \tau_h = -(C_D + k)\overline{u^2}$$

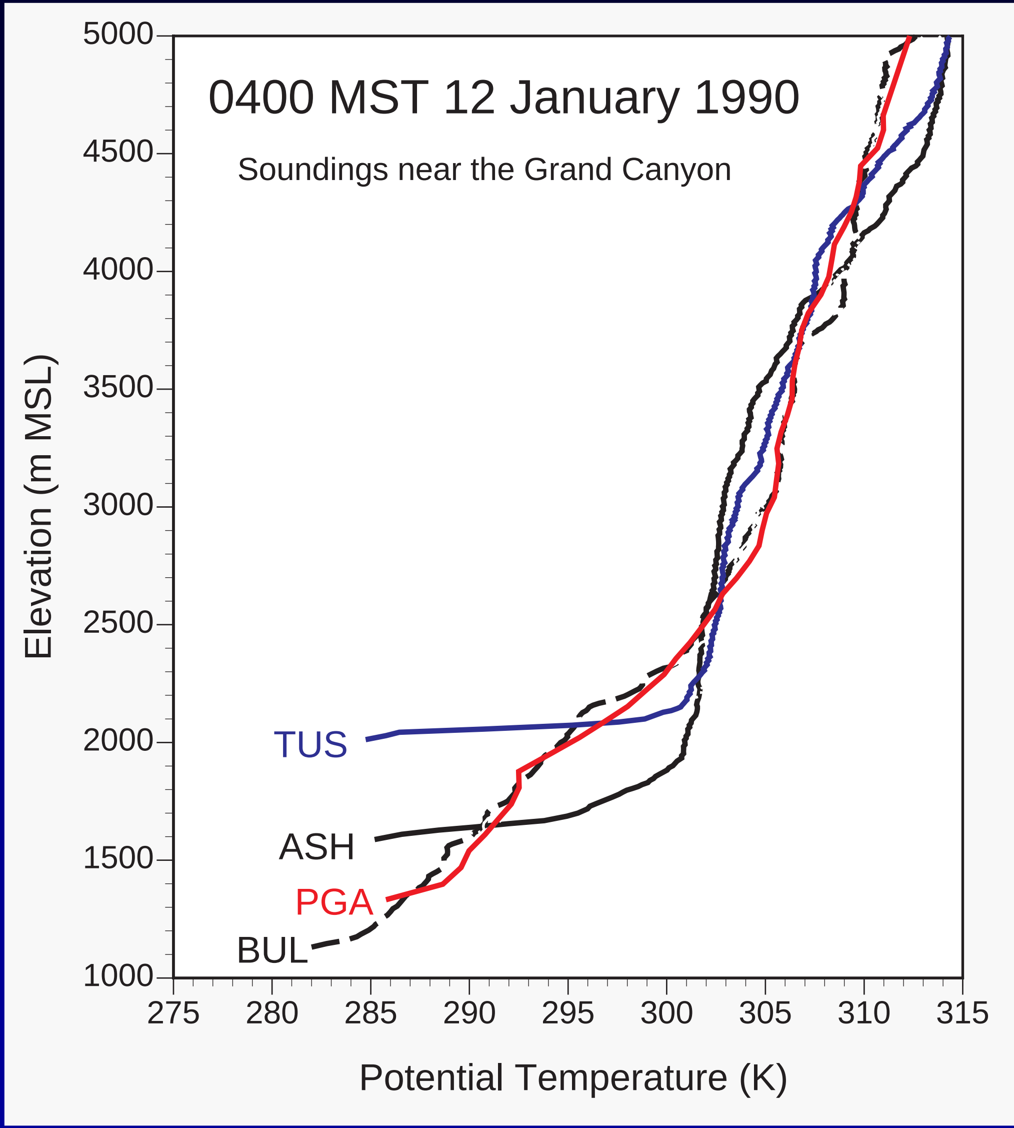
- ADV, BUOY, THERM, KIN.ENTR can be evaluated from VTMX data
- STOR, DRAG can be roughly estimated from VTMX data:

STOR ≤ 25-50 m3/s2, of either sign

DRAG ≈ 25-100 m3/s2, negative

KIN.ENTR ≤ 50-100 m3/s2, positive if flow aloft is downslope

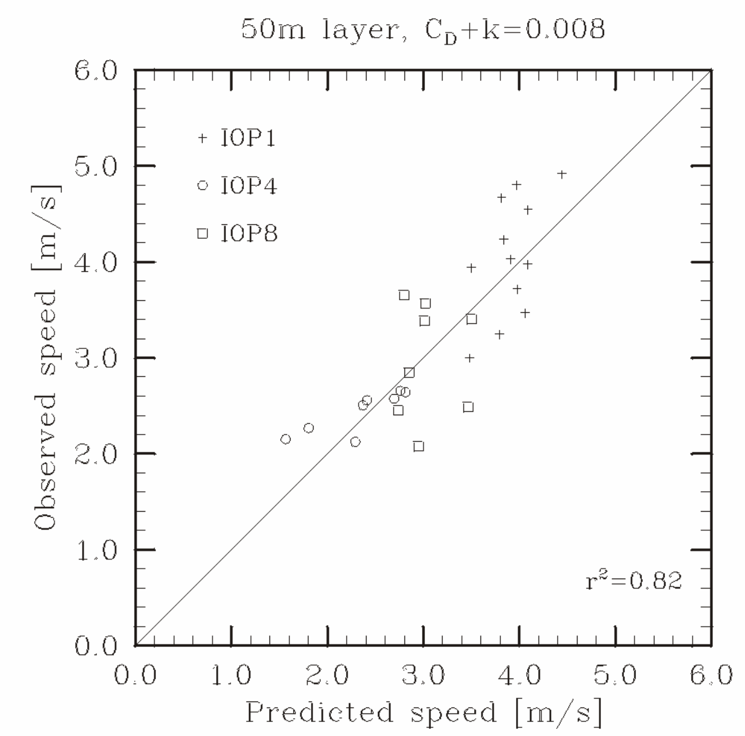
Thermal wind term counter example



Local equilibrium: buoyancy balances drag

Defant (1933)
Prandtl (1942)
Petkovsek and Hocevar (1971)
Munro and Davies (1977)
Manins and Sawford (1979b)
Kondo and Sato (1988)

$$\frac{g}{\theta_0}\sin\alpha\, h\overline{\theta} = (C_D + k)\overline{u^2}$$



- Explains variations between IOPs
- Does not explain variations within IOPs (in space or time)

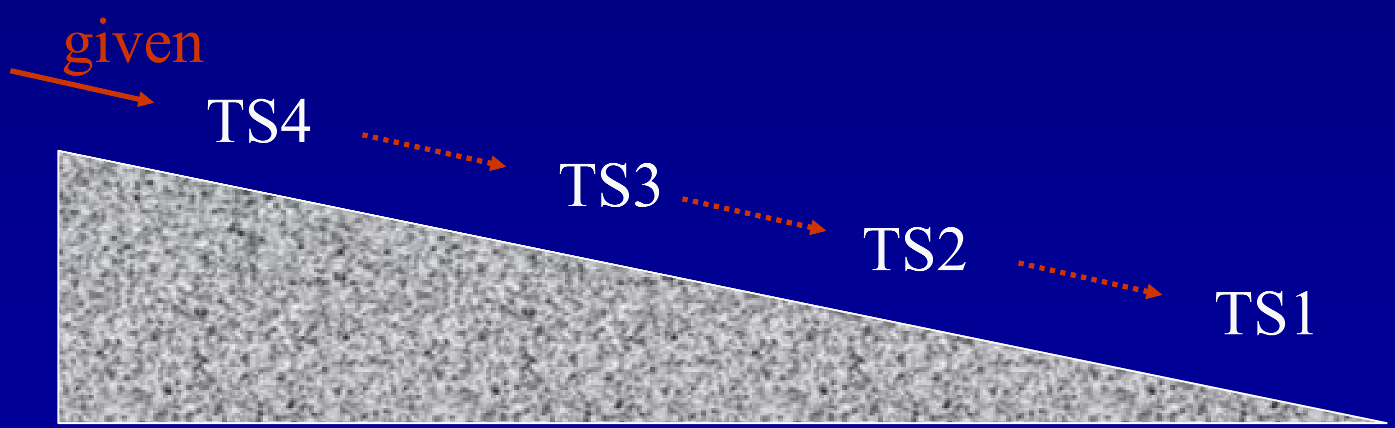
Plot is for different times and different tethersondes within IOPs

Along-slope flow evolution

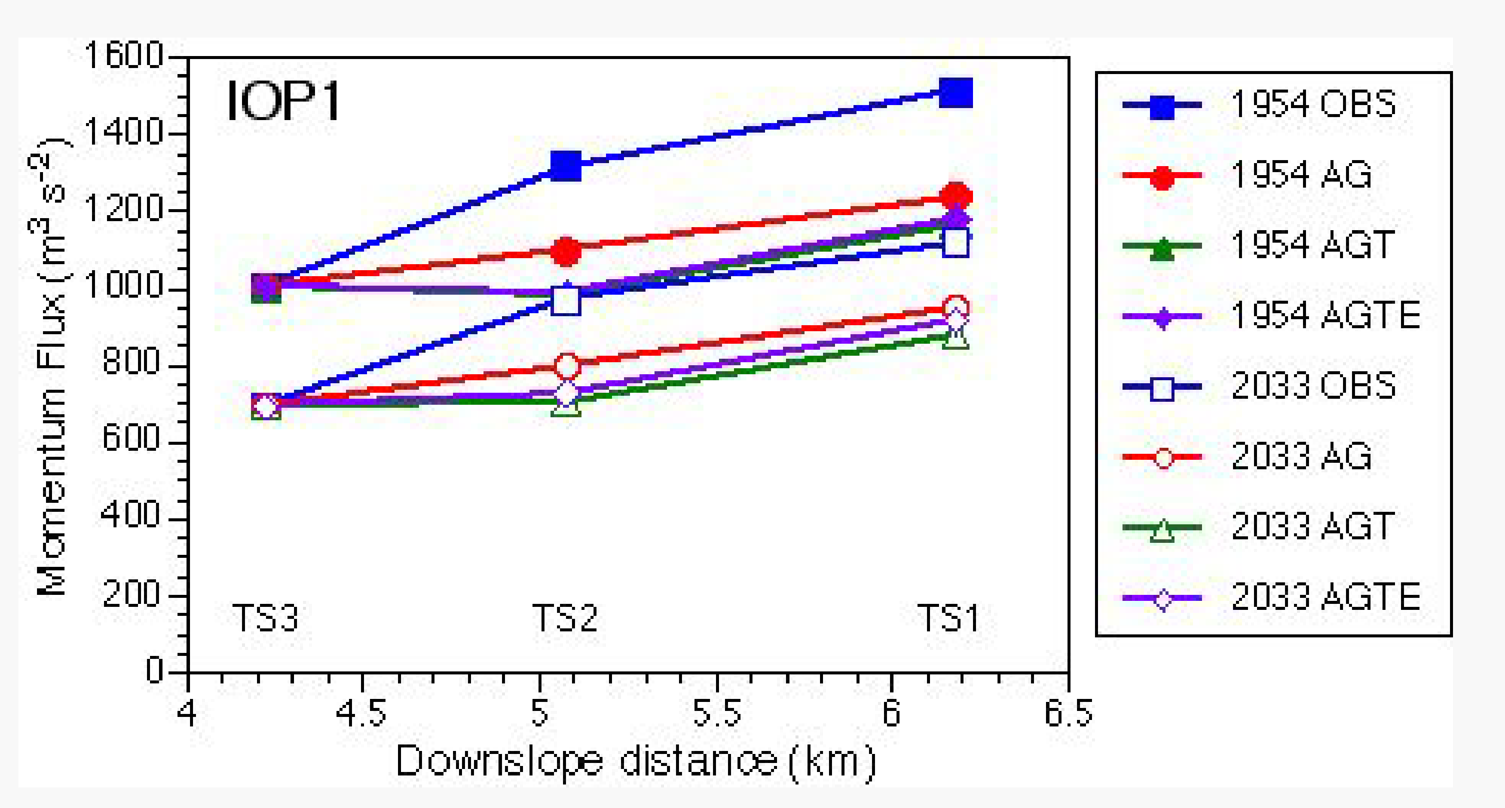
$$\underbrace{\frac{\partial}{\partial t} h\bar{u}}_{STOR} + \underbrace{\frac{\partial}{\partial x} h\overline{u^2}}_{ADV} = \underbrace{\frac{g}{\theta_0}\sin\alpha\, h\bar{\theta}}_{BUOY} + \underbrace{-\frac{g}{\theta_0}\cos\alpha \frac{\partial}{\partial x} h^2\bar{\bar{\theta}}}_{THERM} + \underbrace{\tau_0 - \tau_h}_{DRAG} + \underbrace{-(w\ u)_h}_{KIN.ENTR}$$

Advective-gravity flow

- Businger and Rao (1965)
- Upper limit on along-slope momentum increase

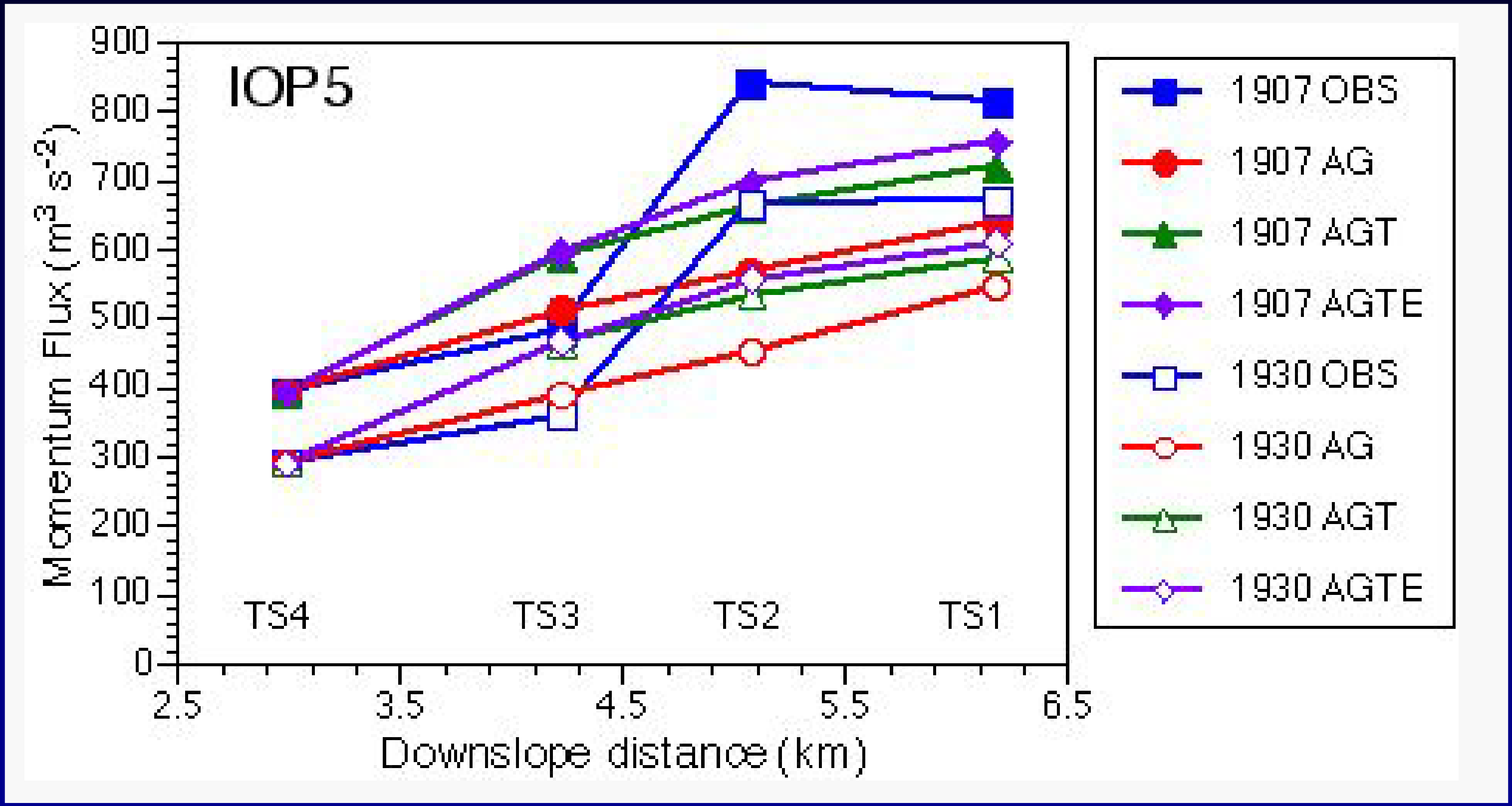


Along-slope flow evolution - IOP1



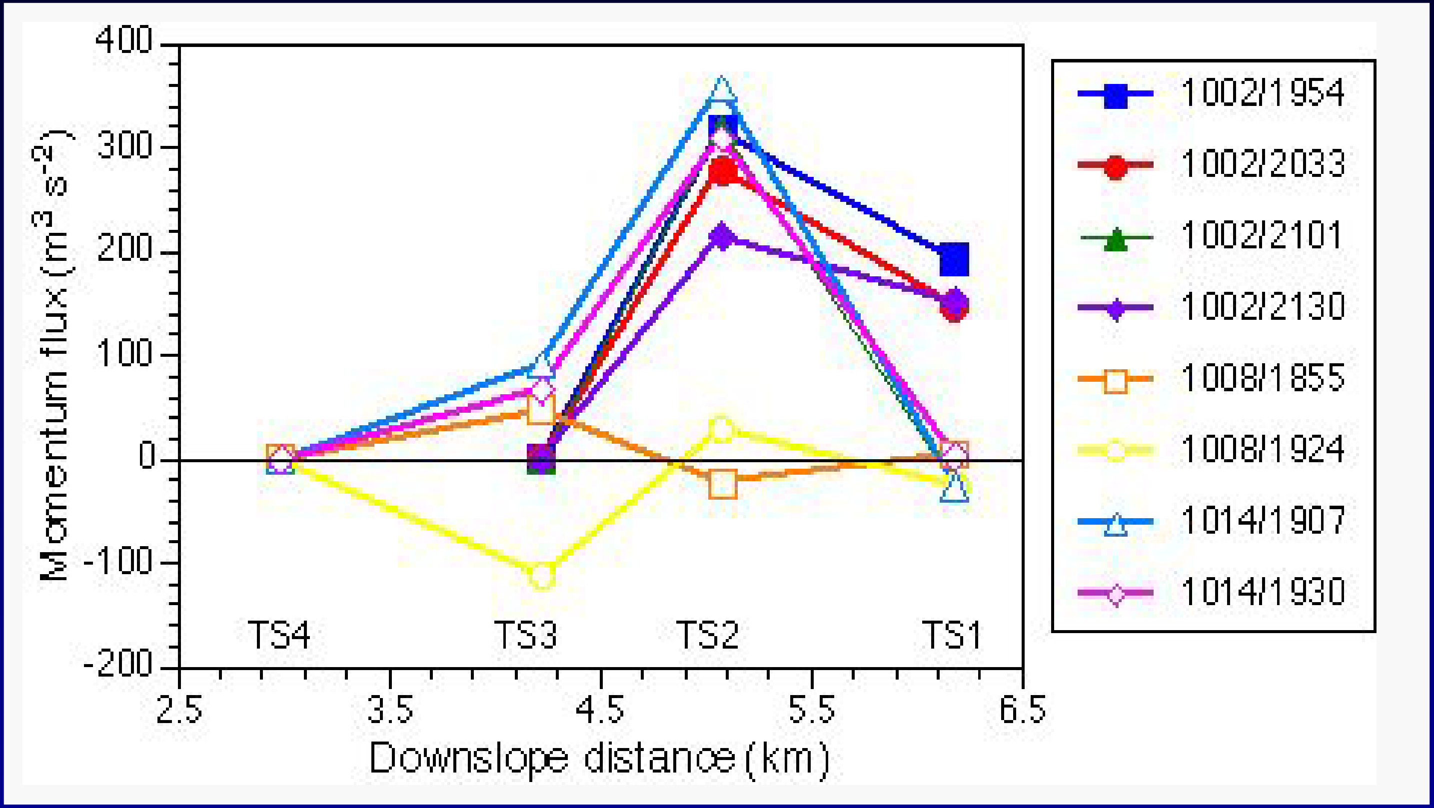
Observed buoyancy is too small to explain the observed momentum flux increase along the slope

Along-slope flow evolution - IOP5



Observed buoyancy is too small to explain the observed momentum flux increase along the slope

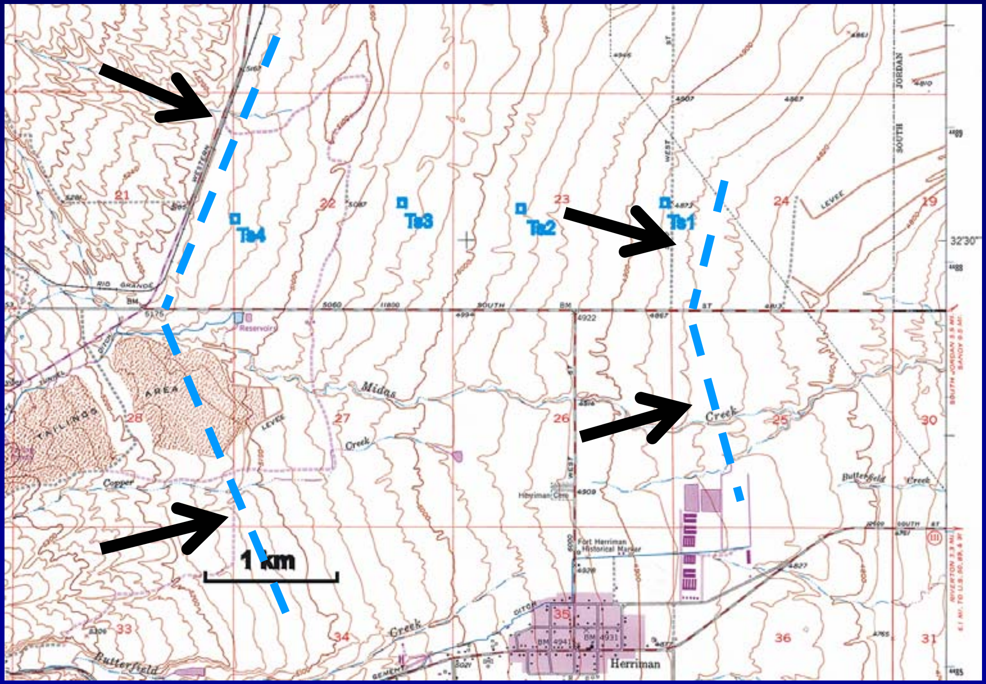
Summary - observed along-slope momentum variations



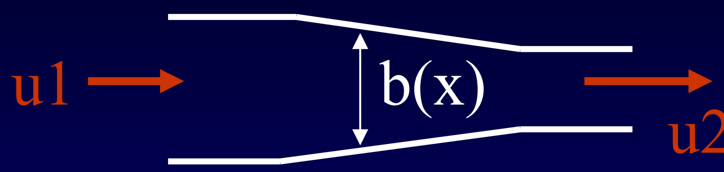
Systematic, strong increase from TS3 to TS2 in weakly stable (2.5 K/km) IOPs 1 and 5 on 10/02 and 10/14. No systematic increase in strongly stable (10 K/km) IOP 4 on 10/08.

Missing terms in the momentum budget

- Coriolis force: too weak to play a role
- Non-stationarity: works in opposite direction in early evening (is small)
- Convergence/confluence effects?



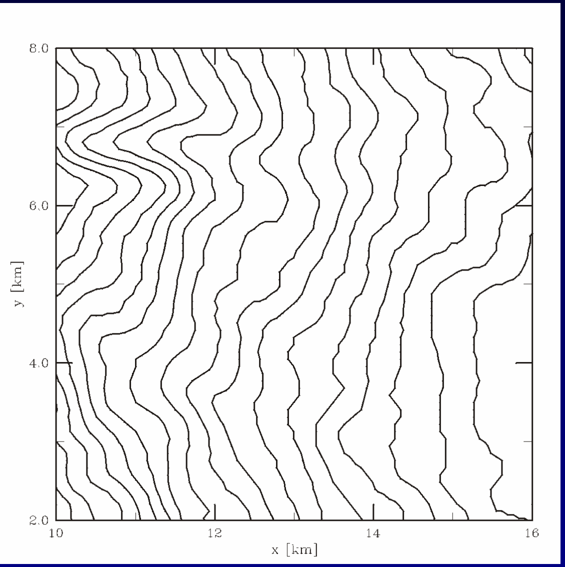
Convergence/confluence effects

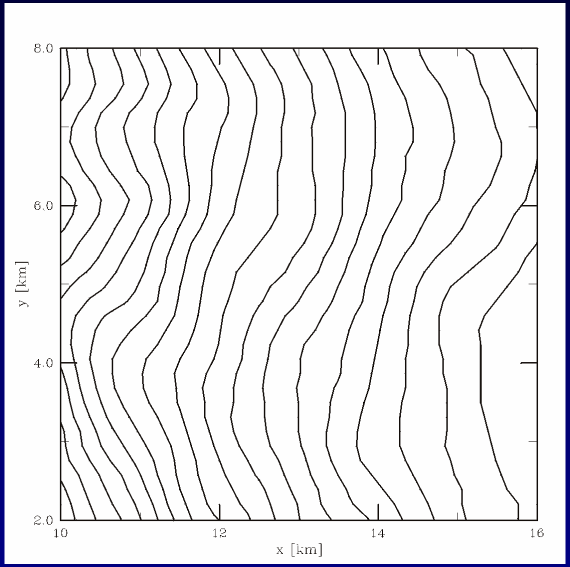


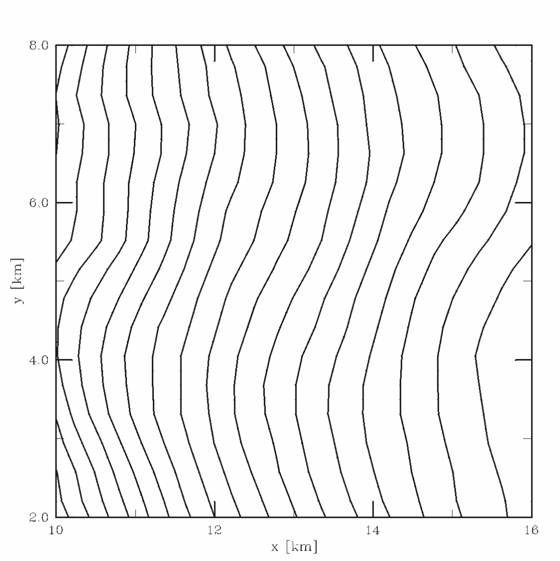
$$\underbrace{\frac{\partial}{\partial t} h\bar{u}}_{STOR} + \underbrace{\frac{\partial}{\partial x} h\overline{u^2}}_{ADV} = \underbrace{\frac{g}{\theta_0} \sin\alpha\, h\bar{\theta}}_{BUOY} - \underbrace{\frac{g}{\theta_0} \cos\alpha \frac{\partial}{\partial x} h^2 \bar{\bar{\theta}}}_{THERM} + \underbrace{\tau_0 - \tau_h}_{DRAG} + \underbrace{-(w\ u)_h}_{KIN.ENTR} \quad \underbrace{-\frac{1}{b}\frac{db}{dx} h\overline{u^2}}_{CONF}$$

- CONV term can cause acceleration and/or deepening
- Acceleration fully materializes only if there is no flow deepening
- VTMX slope: CONV could increase momentum flux by ~50% and more
- But this percentage is strongly scale-dependent!

Scale dependence of downslope direction





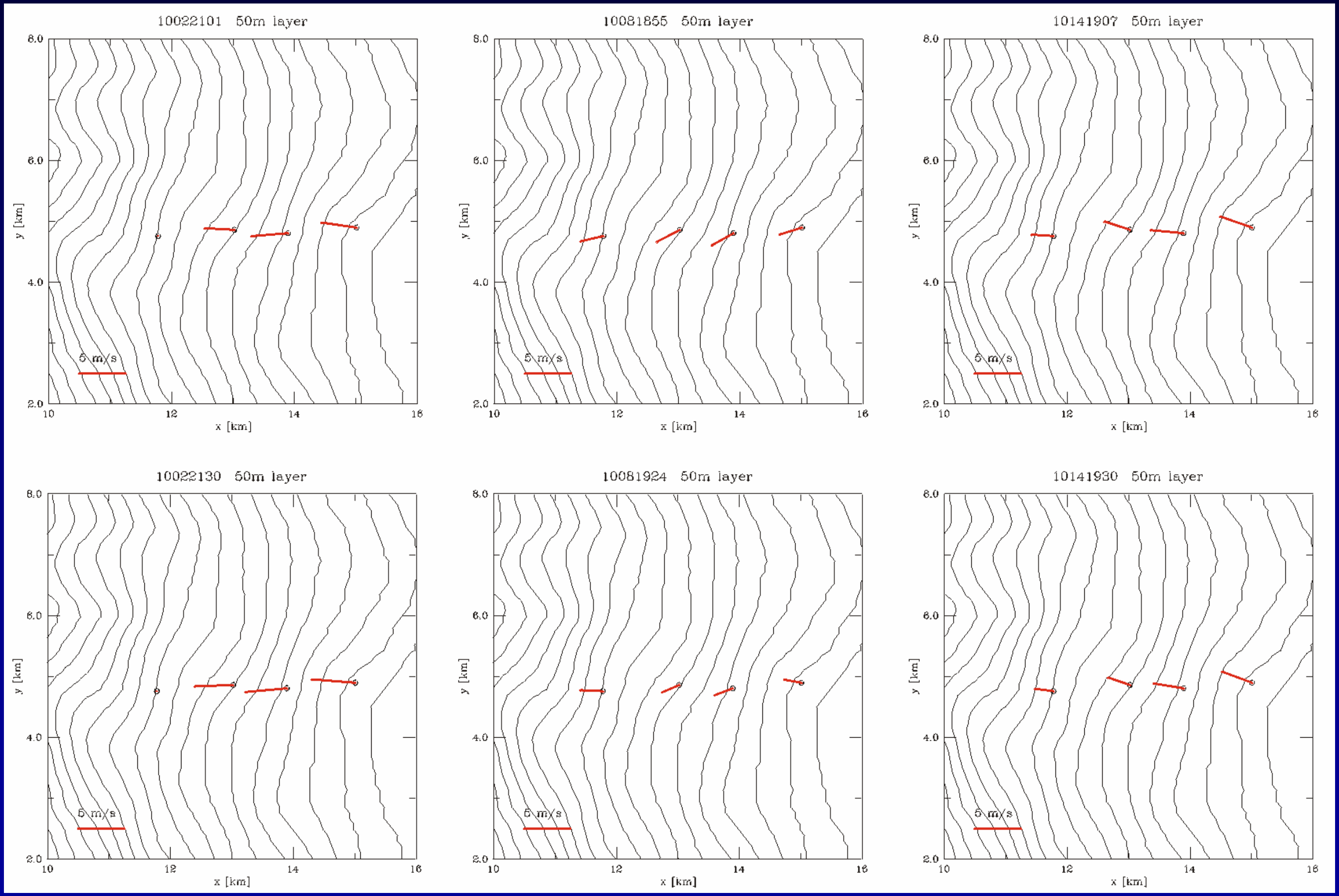


How local is 'local'?

Numerical models: scale cutoff determined by choice of horizontal resolution and horizontal diffusion

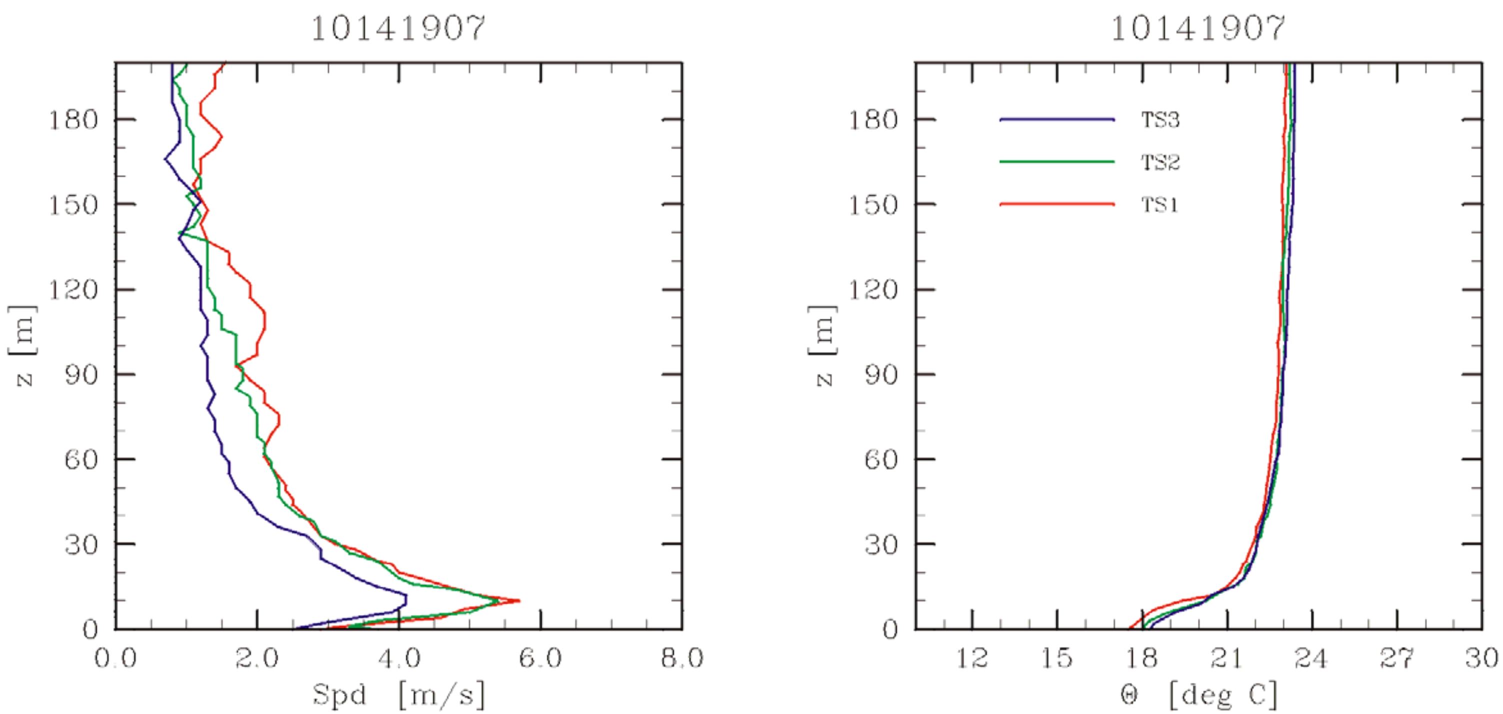
Observed flow direction

IOP1 IOP4 IOP5



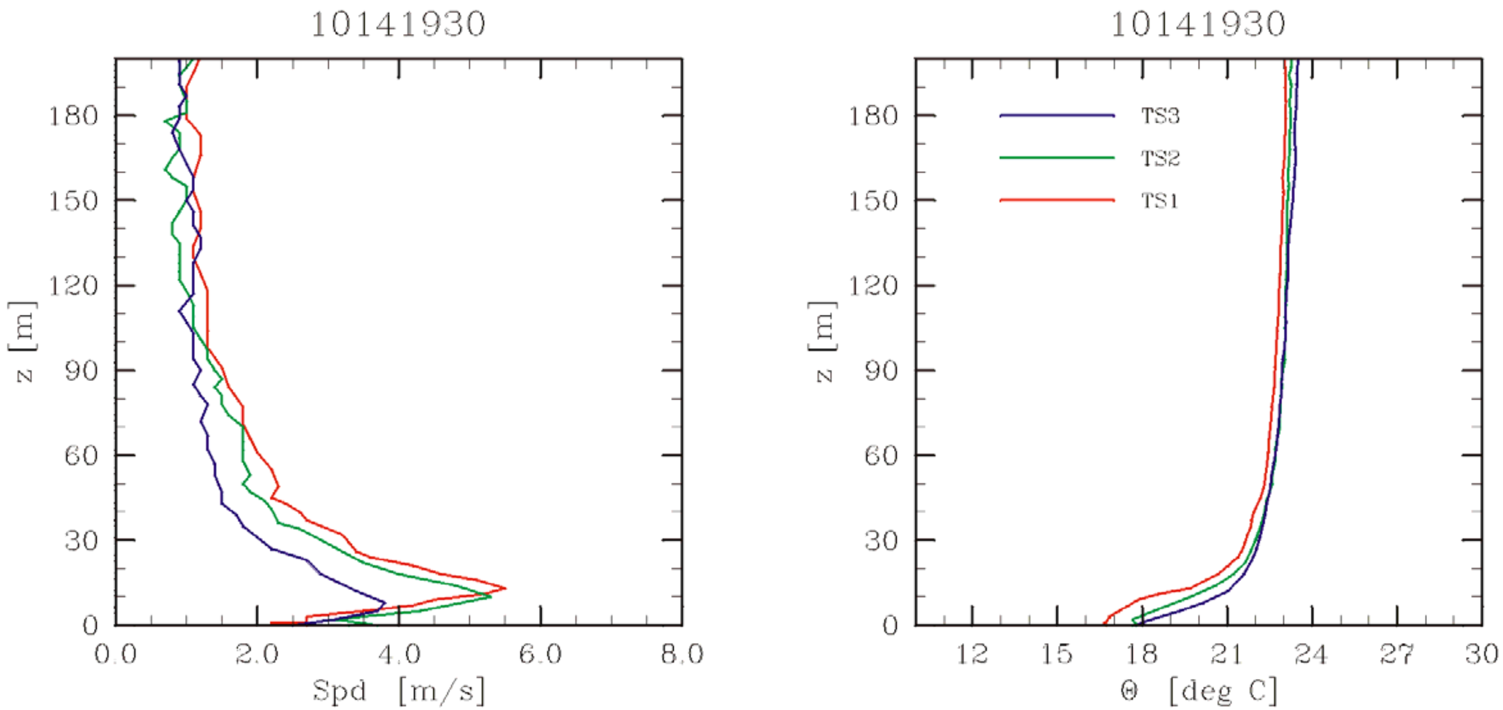
Flow speed-up vs. flow deepening

IOP5



Flow speed-up vs. flow deepening

IOP5



Vertical structure of the flow

Prandtl solution

$$\Delta\theta(z) = \Delta\theta_0 \exp(-z/l)\cos(z/l)$$

$$u(z) = u_0 \exp(-z/l)\sin(z/l) \qquad \textit{where } u_0 = \Delta\theta_0 \sqrt{\frac{g}{\theta_0}\left(\frac{d\theta_0}{dz}\right)^{-1}\frac{K_h}{K_m}}$$

$$l^4 = \frac{4K_h K_m}{N^2 \sin^2\alpha} \qquad \textit{vertical length scale}$$

$$z_{\max} = \frac{\pi}{4} l \qquad \textit{jet height}$$

$$u_{\max} = u_0 \exp(-\pi/4)/\sqrt{2} \approx 0.322 u_0 \qquad \textit{jet speed}$$

Jet speed prediction (Prandtl solution)

	UOBS (m/s)	UMOD (m/s)	ZMAXOBS (m)	K (m2/s)	$\Delta\theta_0$ (K)	$d\theta/dz$ (K/km)
IOP1	4-6	9-10	15		7-8	2.5
IOP4	3-4	3-4	10	0.02-0.06	5-6	10.0
IOP5	3-5	5-6	13		4-5	2.5

- Correct jet speed prediction only in stable case (IOP4)
- Very low eddy diffusivity implied!

Bulk heat equation

$$\underbrace{\frac{\partial}{\partial t} h\bar{\theta}}_{STOR} + \underbrace{\frac{\partial}{\partial x} h\overline{\theta u}}_{ADV} = \underbrace{-h(\bar{u}\sin\alpha - \bar{w}\cos\alpha)\frac{d\theta_0}{dz}}_{BASIC\ STATE\ ADV} + \underbrace{F_0 - F_h}_{TURB\ FLUX\ DIV} + \underbrace{R_o - R_h}_{RAD\ FLUX\ DIV}$$

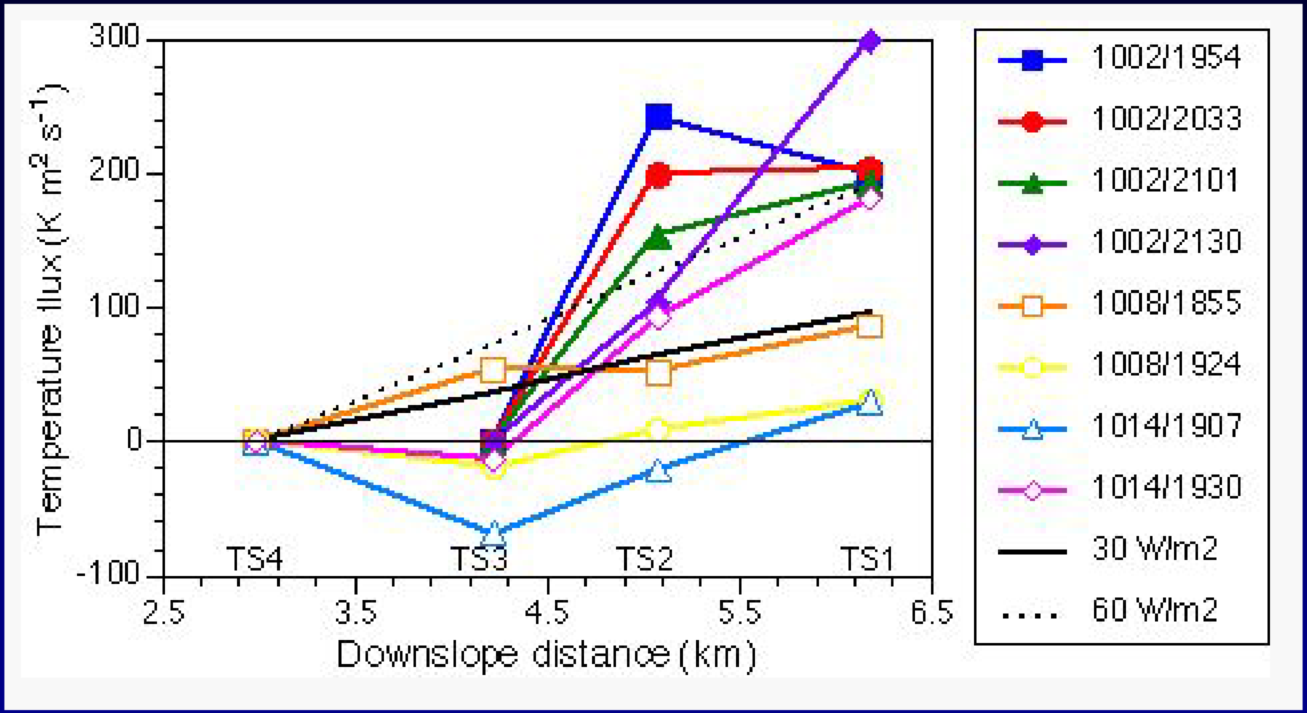
Underlying assumptions

- Plane slope, no crosswind

Common further simplification

- Radiative flux divergence neglected (or absorbed into F)

Observed along-slope heat-flux variations



Increase of downslope flux of temperature deficit from TS3 to TS2 exceeds what would be possible with surface cooling rates of 30 or 60 W/m2 in between the two sites! This is a robust result (a) insensitive to the definition of katabatic layer height, and (b) independent of the definition of the ambient temperature profile (because we are looking at differences between sites only).

Further work

- Acceleration due to concave topography: scale?

→ Assume topo-downgradient flow, apply to range of terrain scales (smoothing)

- What determines jet max height and strength?

→ Prandtl model with K(Ri) feedback

- Explain observations at TS4 (uppermost site)

→ Couple heat & momentum budgets