Subfilter-scale modeling for ARPS
[following the trail to the log law behavior]

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Outline

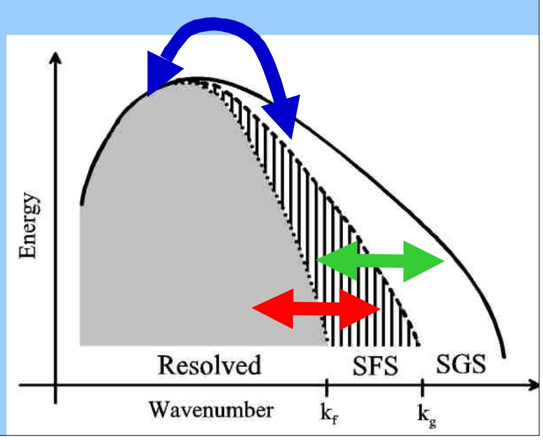
Velocity estimation models: leading to the mixed model of Chow

Fork in the Road: from smooth to rough walls: searching for Andren's Grail

Modified eddy viscosity:
- Sullivan; Ding/Arya
- Porte-Agel model
- Kosovic′ model
- Extended boundary condition

SFS & SGS scale partitioning:
A way of thinking
Ideas suggested, e.g., by Carati, et al. (2001), Zhou, et al. (2001), and Collis (2001)

- Resolved scales
  - Scales on grid after filtering
- Subfilter scales (SFS)
  - can be recovered by several methods
- Subgrid scales (SGS)
  - must model



An approach to SFS modeling by Velocity Estimation

SFS stress $\tau_{ij} = \overline{u_i u_j} - \overline{u}_i \overline{u}_j$

- Velocity estimation

$$u = f(\overline{u}) \rightarrow u = \overline{u} + \cdots$$

$$u_i = \overline{u}_i - \frac{\Delta^2}{24}\nabla^2 \overline{u}_i + O(\Delta^4)$$

$$\tau_{ij} = \overline{u_i u_j} - \overline{u}_i \overline{u}_j$$

NB: All filtering done at one scale that is greater than the grid scale.

SFS stress model

[Satisfies full τ_{ij} evolution equations to order of expansion]

$$\begin{aligned}\tau_{ij} &= \overline{\overline{u}_i \overline{u}_j} - \overline{\overline{u}}_i \overline{\overline{u}}_j - \frac{\Delta_f^2}{24}\overline{\overline{u}_i \nabla^2 \overline{u}_j} - \frac{\Delta_f^2}{24}\overline{\overline{u}_j \nabla^2 \overline{u}_i} \\ &+ \frac{\Delta_f^2}{24}\overline{\overline{u}}_i \nabla^2 \overline{\overline{u}}_j + \frac{\Delta_f^2}{24}\overline{\overline{u}}_j \nabla^2 \overline{\overline{u}}_i + O(\Delta^4)\end{aligned}$$

Equivalent to the modified Clark model [which is used later and also called the tensor diffusivity model]:

$$\tau_{ij} \approx \frac{\Delta_f^2}{12}\overline{\frac{\partial \overline{u}_i}{\partial x_m}\frac{\partial \overline{u}_j}{\partial x_m}} + O(\Delta^4)$$

Walls and Boundaries

There is a great deal of literature on boundary effects; I am not going to touch on that broad domain, but rather move now to some thoughts about how rough boundaries impact atmospheric wall layers. The SFS & SGS models are critical for atmospheric boundary layers because of

- High Reynolds number and rough boundary.
- Near-wall energy-containing eddies not resolved.
- It is argued that the errors from the near surface affect entire boundary layer.

There is also impressive evidence now about the expected behavior near the rough wall in the atmosphere provided by field experiments.

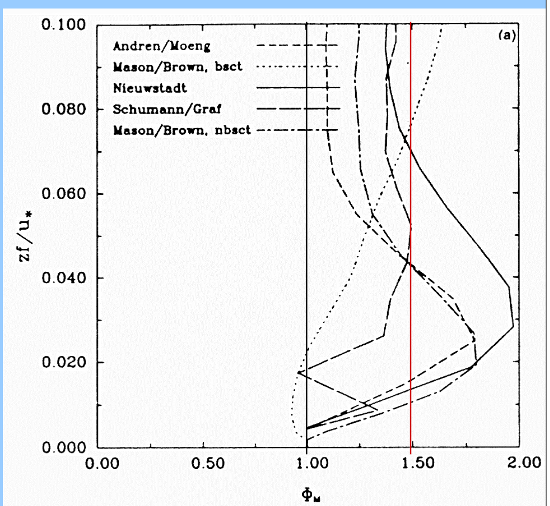
The Fork in the Road

- Andren,et al. (1994) undertook an examination of four LES simulation codes and in some sense thereby created a "Holy Grail" for rough wall flows; namely, does the mean velocity profile follow similarity theory in the neutral boundary layer, i.e, do we get a log profile?

- In particular we look for

$$\Phi = \frac{\kappa z}{u_*}\sqrt{\left(\frac{\partial < \overline{u} >}{\partial z}\right)^2 + \left(\frac{\partial < \overline{v} >}{\partial z}\right)^2}$$

to be unity in the near wall region [say first 10 % of the layer depth].



The Fork in the Road – Various approaches

- Sullivan, McWilliams & Moeng (1994) proposed a two-part eddy viscosity to improve the agreement between the LES and similarity solutions:

$$\tau_{ij} = -2\nu_t \gamma S_{ij} - 2\nu_T \langle S_{ij} \rangle$$

$$\gamma = S' / [S' + \langle S \rangle]$$

Code is based on Moeng (1984): pseudo-spectral in horizontal and FD in vertical.

- Ding, Arya & Lin (2001) made some modifications to the mean eddy viscosity of this model and achieved some additional improvements.

- Porte-Agel, Meneveau and Parlange (2000) recognized that for atmospheric flows the assumption that the dynamic model coefficient C would be invariant over the scales between the test and grid filters would fail. They employed a second test filtering operation and so determined, in essence, three values of C at the grid filter, first test filter and second test filter scales, which scales differed by a factor of 2.

Near the wall they reduce Φ to about 1.2 or less

The Fork in the Road- A nonlinear SGS model

Kosovic′ (1997)

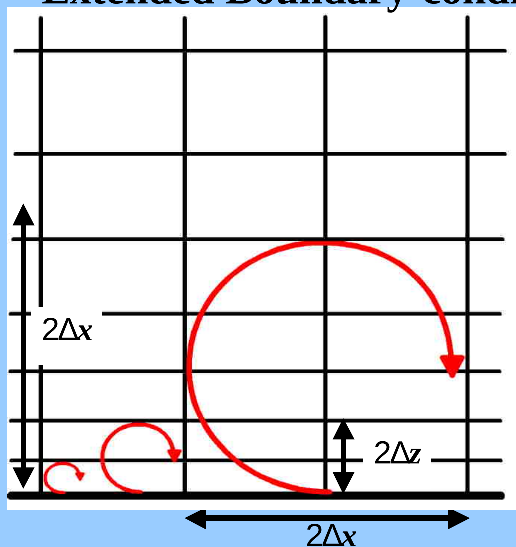
- Kosovic′ (1997) created a nonlinear SGS model based on a general nonlinear constitutive relationship. Whereas linear SGS models have one model parameter [e.g., the Smag. Constant], this model has 3 [which are not, however, independent. In the end, a backscatter parameter, C_b, is determined by numerical experiment. Insight, since backscatter parameter is > 0, the appropriate Smag. constant is greater than one would expect.

Code is based on Moeng (1984).

Near the wall $0.8 < \Phi < 1.1$

The Fork in the Road-

Extended Boundary-condition approaches



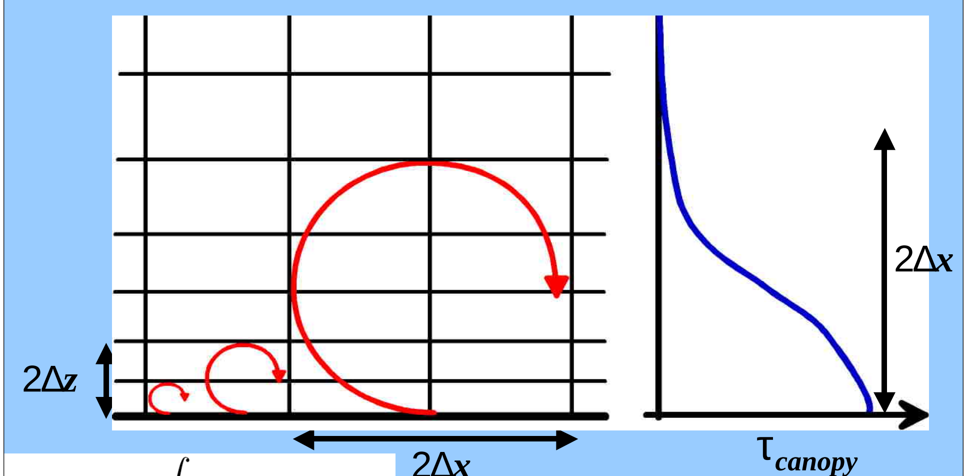
Insights from Kaltenbach (1998), *CTR Annual Briefs*:

Competing requirements:

- **Wall-normal spacing has to be reasonably fine to allow meaningful representation of "mean" flow profiles.**
- **Horizontal spacing defines the size of the smallest turbulent eddies to be resolved. "Thickness" of the near wall zone should correspond roughly to average horizontal spacing.**

Leads here to "pancake" grids And special wall modeling.

Canopies: **Brown, Hobson & Wood (2001) borrowed the idea of the canopy effect in vegetation to represent the additional SGS turbulent transfer that is missing near the wall. We have incorporated that idea into ARPS for the study of valley winds.**



$$\tau_{i,can} = -\int C_c a(z)|\overline{u}|\overline{u}_i dz$$

F. K. Chow:
Neutral flow
LES: Mod. Clark + Smag. + Canopy

Note: filter size $\Delta = 2\Delta_x$

Combined SFS and SGS model

$$\tau_{ij} = \tau_{SFS} + \tau_{SGS}$$

- **SFS = Series/Modified Clark**

$$\tau_{ij} \approx \frac{\Delta_f^2}{12} \overline{\frac{\partial \overline{u}_i}{\partial x_m} \frac{\partial \overline{u}_j}{\partial x_m}}$$

- **SGS = Smagorinksy + Canopy (Brown et al. 2001)**

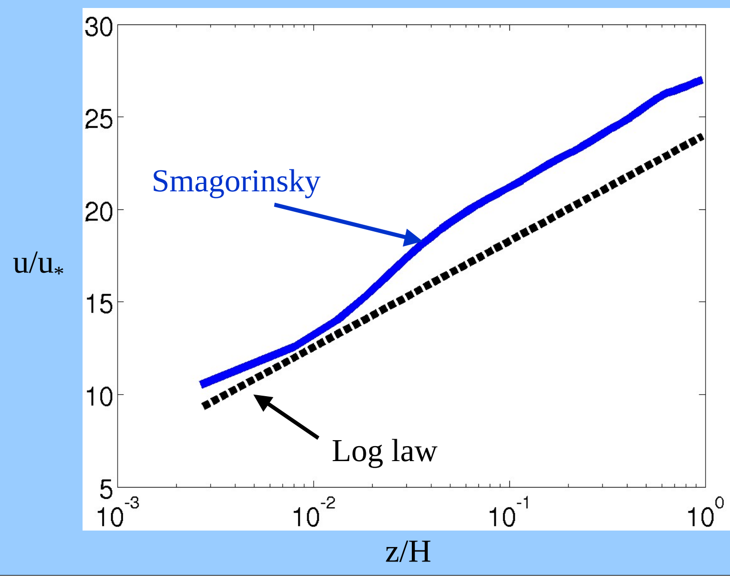
$$\tau_{ij} \approx -2\nu_T \overline{S}_{ij}$$

$$\tau_{i,can} = -\int C_c a(z) |\overline{u}| \overline{u}_i dz$$

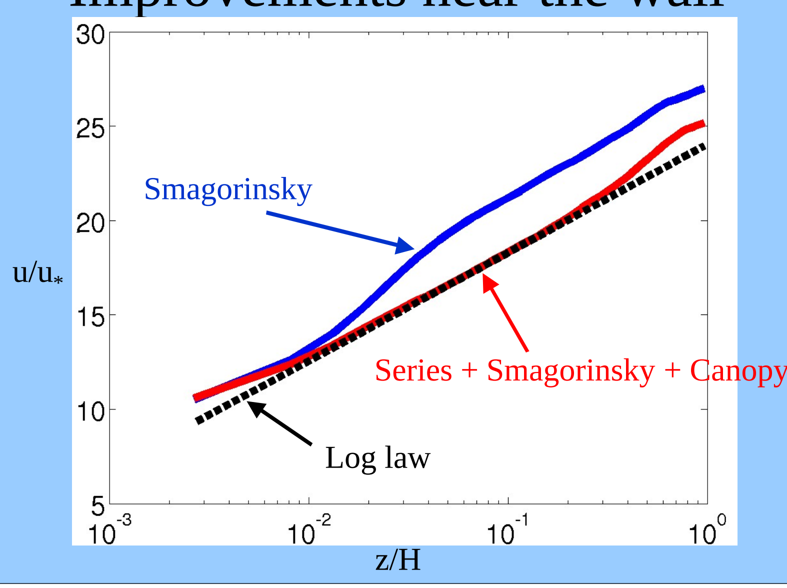
Neutral boundary layer (NBL) tests

- Similar to case done by Andren et al. 1994
- High Reynolds number
- Rough wall – viscous layer not resolved
  - Surface flux – log law
- Low resolution
  - 40 x 40 x 40 grid
  - 1.3 km x 1.3 km x 1.5 km domain
- Geostrophic balance - “Ekman” spiral
- ARPS code

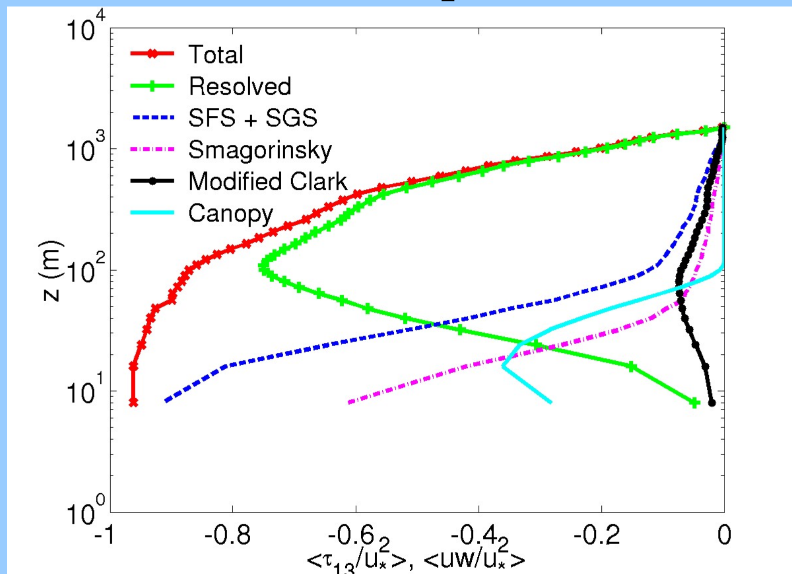
Problems near the wall…



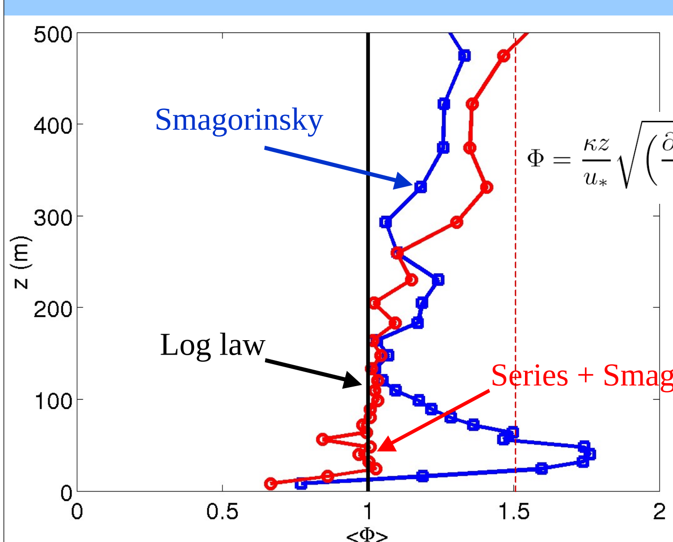
Improvements near the wall



NBL stress profiles



Neutral BL nondimensional shear



$$\Phi = \frac{\kappa z}{u_*}\sqrt{\left(\frac{\partial < \overline{u} >}{\partial z}\right)^2 + \left(\frac{\partial < \overline{v} >}{\partial z}\right)^2}$$