Pooling and Inversion Break Up in Complex Terrain: Modeling and Analysis of VTMX Data

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Goals of EMP/VTMX to be addressed

1. Increase our understanding of Mechanisms Responsible for Vertical Transport and Mixing

   Science questions addressed:

   - Look at dominant balances of downslope flows
   - Look at cold pool destructions

2. Improve our ability to measure quantities required for our understanding

3. Develop improved treatments of vertical transport and mixing for use in conceptual models.

   Science questions addressed:

   - Simple theoretical arguments
   - Test of models
   - Propose new models

ASU Contribution to the VTMX Program

Theoretical Modeling

- Down-slope and up-slope flows
- Transition period
- Initiation of up-slope flow

Laboratory Experiments

- Down-slope (simple slopes)
- Up-slope flows (simple slopes)
- Basin (pooling and slope flows)
- Transition (on a simple slope)

VTMX Field Program (Limited)

- Sonic Anemometers (turbulence)
- Cup anemometers
- Tethered balloons (2) – profiling (T, V)
- Radiation (in and out)
- Aerosol measurements (TEOM/Streaker/Dustrack)

Analysis of PAFEX Data

- Data taken during 1998 (NSF) experiments (January-February, July-September)
- Transition periods

Downslope – Field Observations

Tower

Balloons

Downslope Flow – Governing Equations

Momentum Equation

$$\frac{DU}{Dt} \equiv \frac{\partial U}{\partial t} + U\frac{\partial U}{\partial \hat{x}} + W\frac{\partial U}{\partial \hat{z}} \approx -\alpha_H \Delta b + \partial \tau / \partial \hat{z}$$

Where

$$\Delta b = -g\,\Delta\theta / \bar{\theta}(0)$$ ($\Delta b < 0$ for positively buoyant or less dense fluid)

and

$$\Delta\theta = \theta(\hat{z}) - \theta_E(\hat{z} = h)$$

The temperature difference between the air in the current and in the external region at just outside the boundary layer

Downslope Flow – Governing Equations

Temperature difference (expressed as buoyancy differences)

$$\frac{D\Delta b}{Dt} \equiv \frac{\partial \Delta b}{\partial t} + U\frac{\partial \Delta b}{\partial \hat{x}} + W\frac{\partial \Delta b}{\partial \hat{z}} \approx \frac{\partial F_b}{\partial \hat{z}} - \frac{E_b}{h}\Delta b + \alpha_H N_E^2 U_H$$

Where

$$\partial F_b / \partial \hat{z} = (g / \bar{\theta}(0)) \cdot (\partial F_\theta / \partial \hat{z})$$

($\partial F_\theta / \partial \hat{z}$ being the heat flux divergence at the surface)

$\frac{E_b}{h_H}(\Delta\theta)$ is average rate of mixing with the external fluid

($E_b = dh_H / dt$ being the boundary entrainment velocity)

$-\alpha_H (\partial \theta_E / \partial \hat{z}) U_H$ is the rate of temperature rise of fluid elements caused by their downward displacement (Fleagle 1950)

Downslope Flow – Theoretical Model – Periodicity

Linearized governing equations with neglected flux divergence and the entrainment-rate,

$$\frac{\partial U}{\partial t} \approx -\alpha_H \Delta b + \partial \tau / \partial \hat{z} \qquad \frac{\partial \Delta b}{\partial t} \approx \alpha_H N_E^2 U_H$$

$$\frac{\partial^2 U}{\partial t^2} + \alpha_H^2 N_E^2 U = 0$$

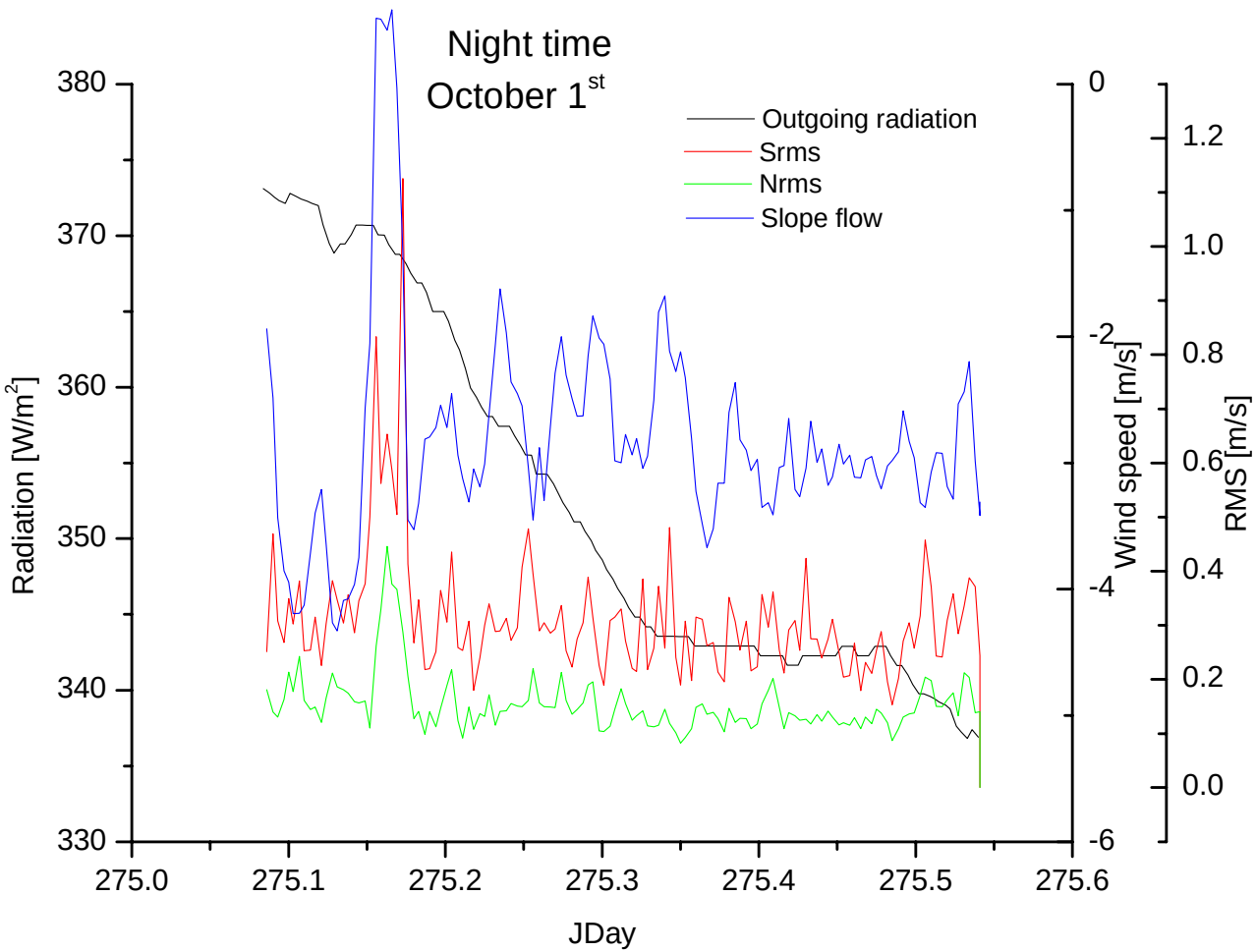
have oscillatory solution with the frequency

$$\omega \sim \alpha_H N_E$$

or period

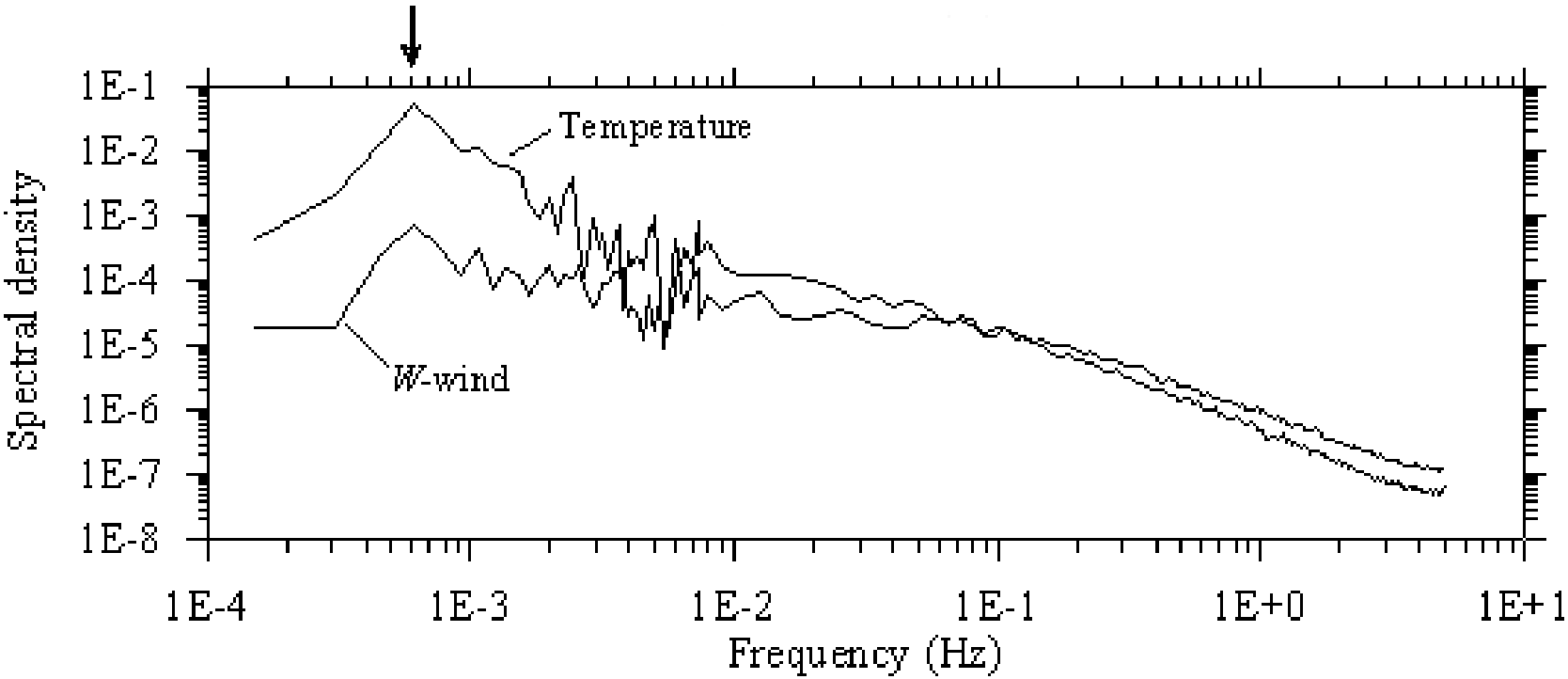
$$T \sim \frac{2\pi}{\alpha_H N_E}$$

Pulsation of downslope flow



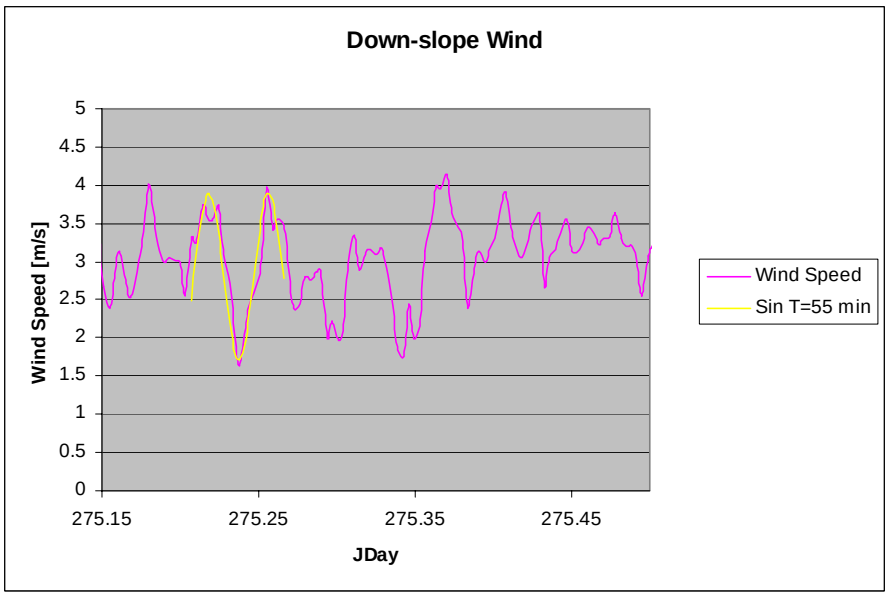
T ≈ 30 min

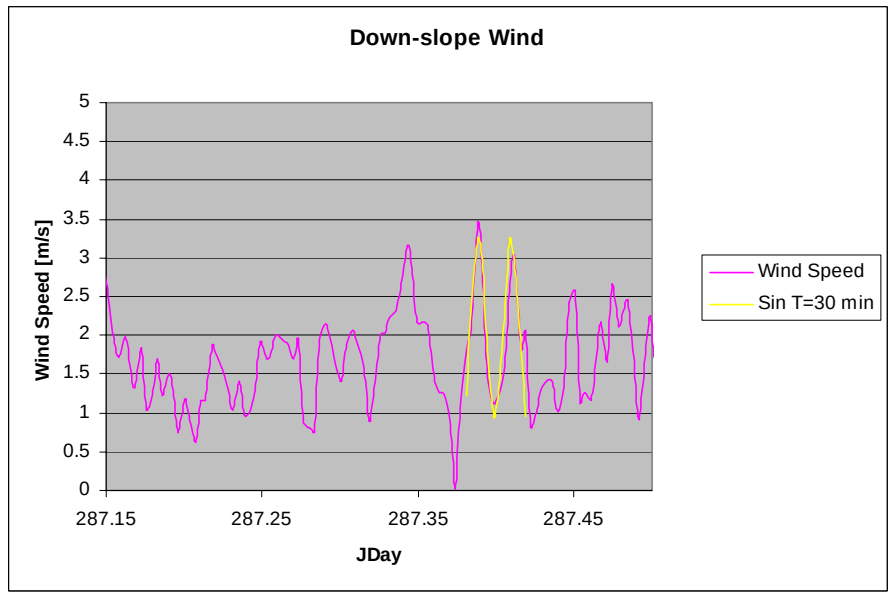
VTMX - Spectra



Peak corresponds to period $T \approx 23\,\text{min}$

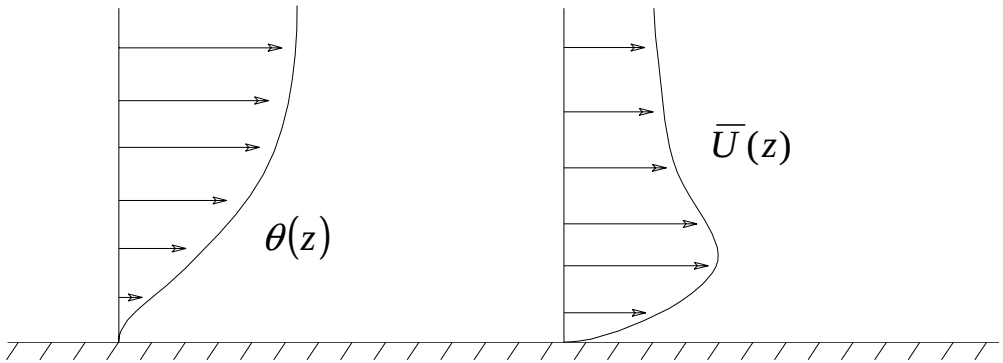
Pulsation of downslope flow





ACS Period varies from 20 to 60 minutes

Parameterization of Vertical Mixing



$$\frac{\partial \overline{q^2/2}}{\partial t} + U_j \frac{\partial \overline{q^2/2}}{\partial x_j} = \underbrace{-\overline{u_i u_j}\frac{\partial U_i}{\partial x_j}}_{\mathrm{P}} + \underbrace{\overline{bw}}_{\mathrm{B}} - \frac{\partial\left(\overline{u_j q^2/2 + \overline{pu_j}}\right)}{\partial x_j} - \underbrace{\varepsilon}_{\varepsilon}$$

Flux Richardson Number

$$R_f = \frac{-\overline{bw}}{-\overline{u_i u_j}\dfrac{\partial \overline{U}_i}{\partial x_j}}$$

Gradient Richardson Number

$$R_{ig} = \frac{N^2}{\left(\dfrac{\partial \overline{U}}{\partial z}\right)^2}$$

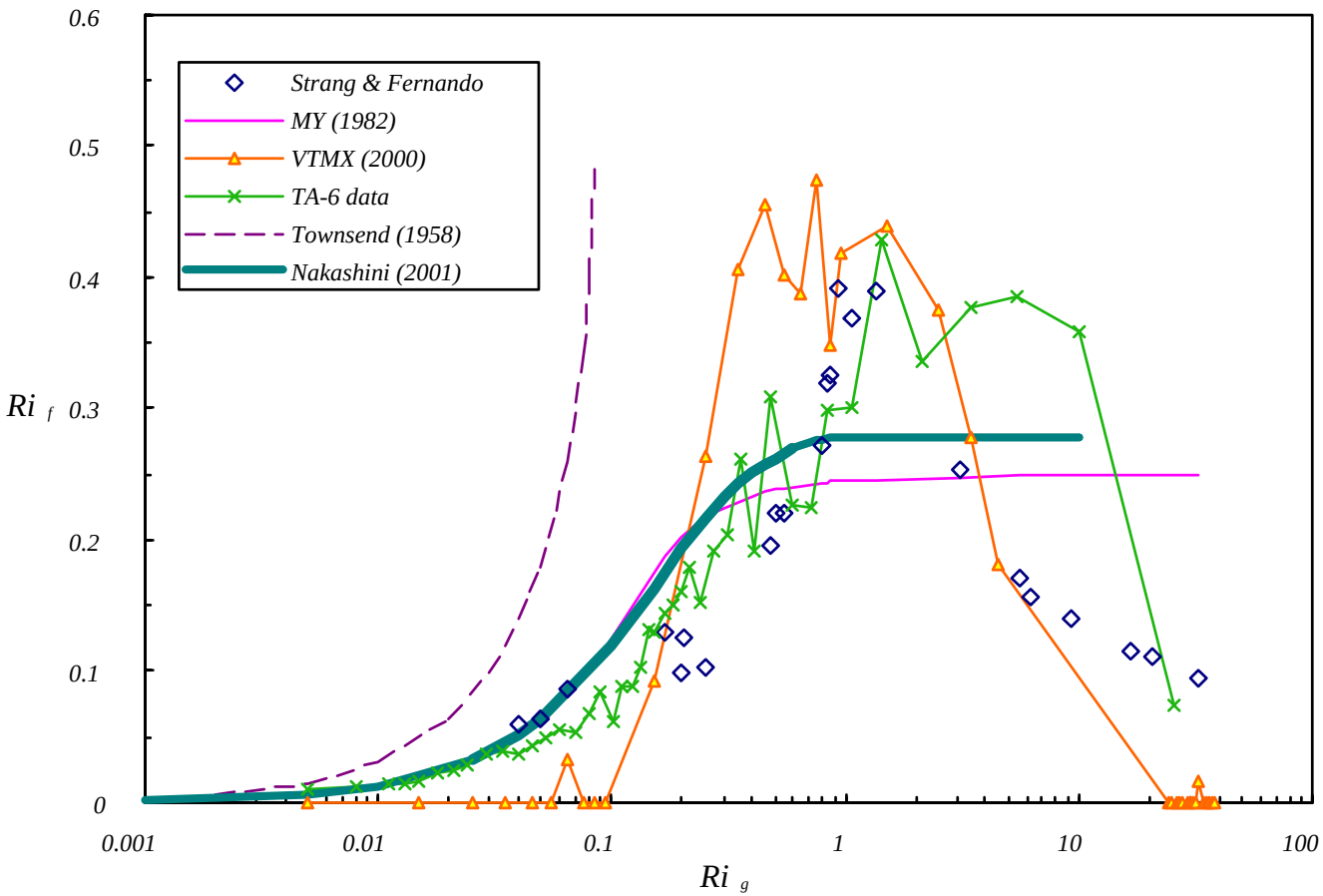
Diffusion Coefficients

$$-\overline{u_i u_j} = K_m\left(\frac{\partial U_i}{\partial x_j} + \frac{\partial U_j}{\partial x_i}\right)$$

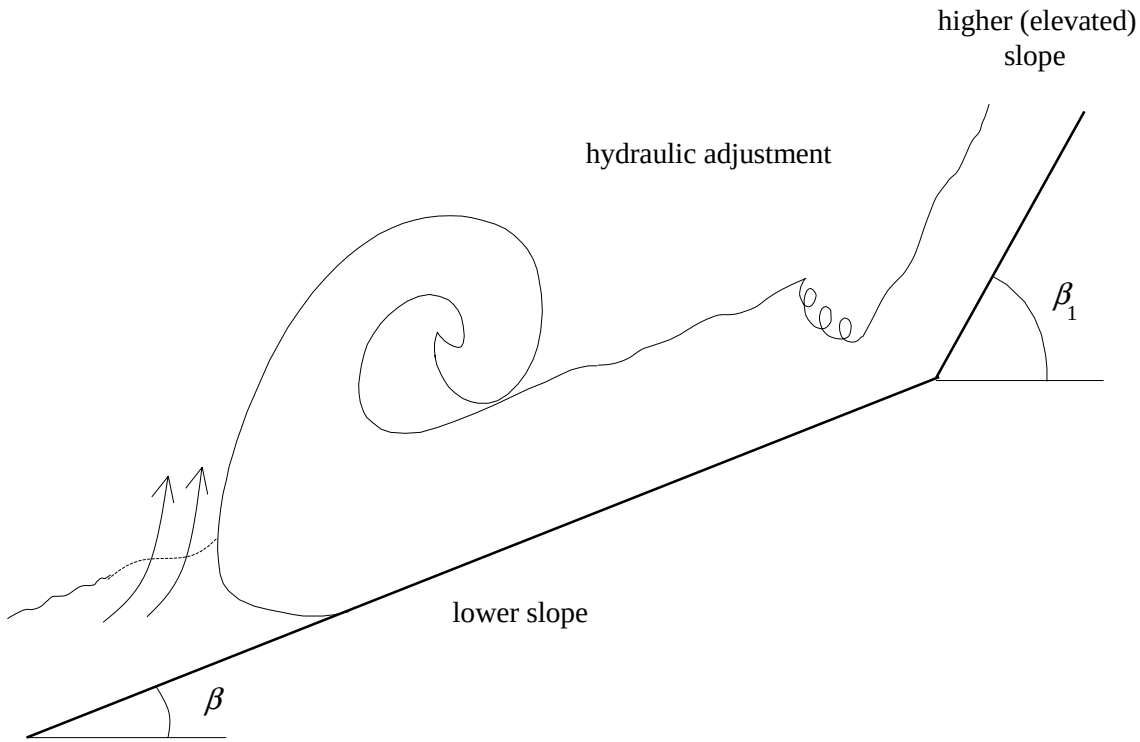
$$-\overline{bu_i} = K_h\left(\frac{\partial \overline{b}}{\partial x_i}\right)$$

eg. $$-\overline{uw} = K_m \frac{\partial \overline{U}}{\partial z}$$

Ri_f vs. Ri_g


Strang & Fernando
MY (1982)
VTMX (2000)
TA-6 data
Townsend (1958)
Nakashini (2001)
0.6
0.5
0.4
0.3
0.2
0.1
0
Ri f
0.001
0.01
0.1
1
10
100
Ri g

Slope Discontinuity

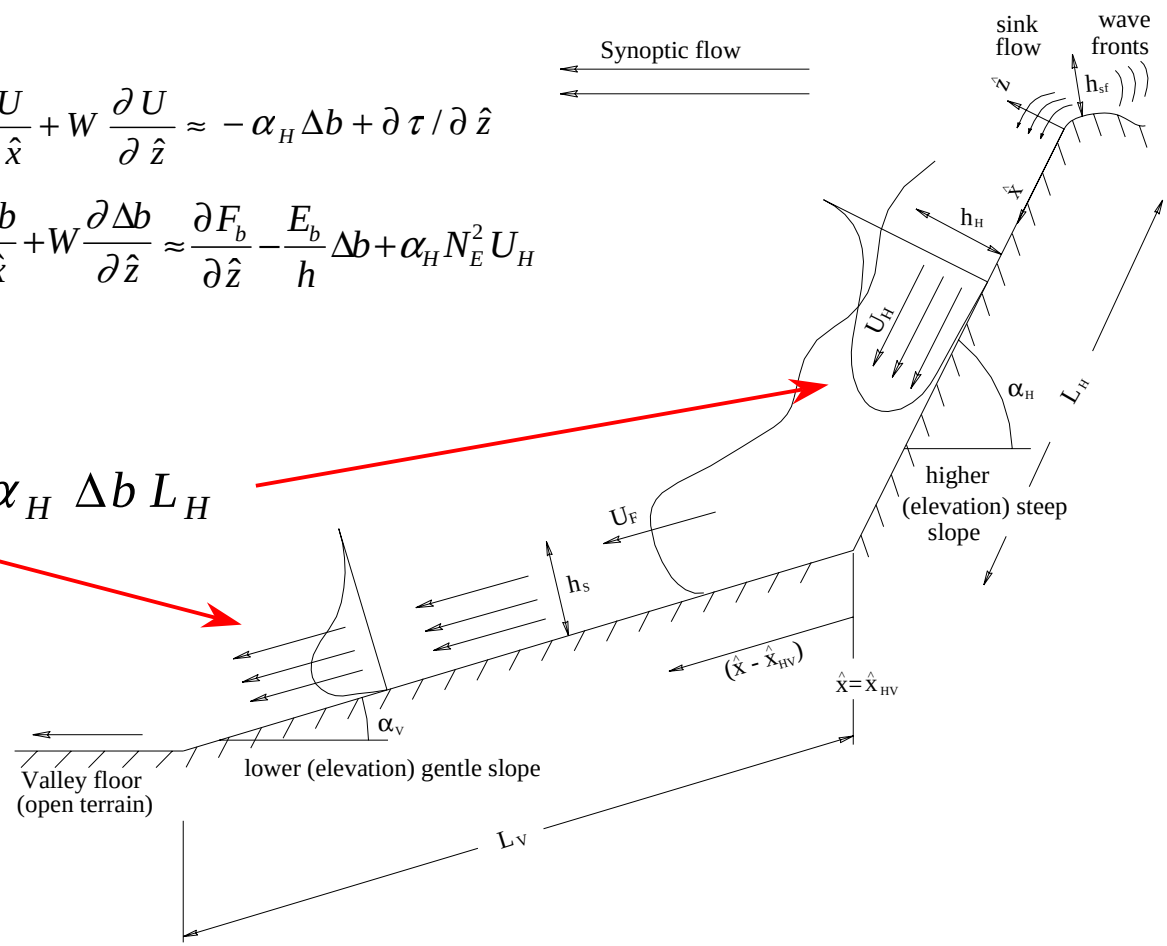


Idealized Topography

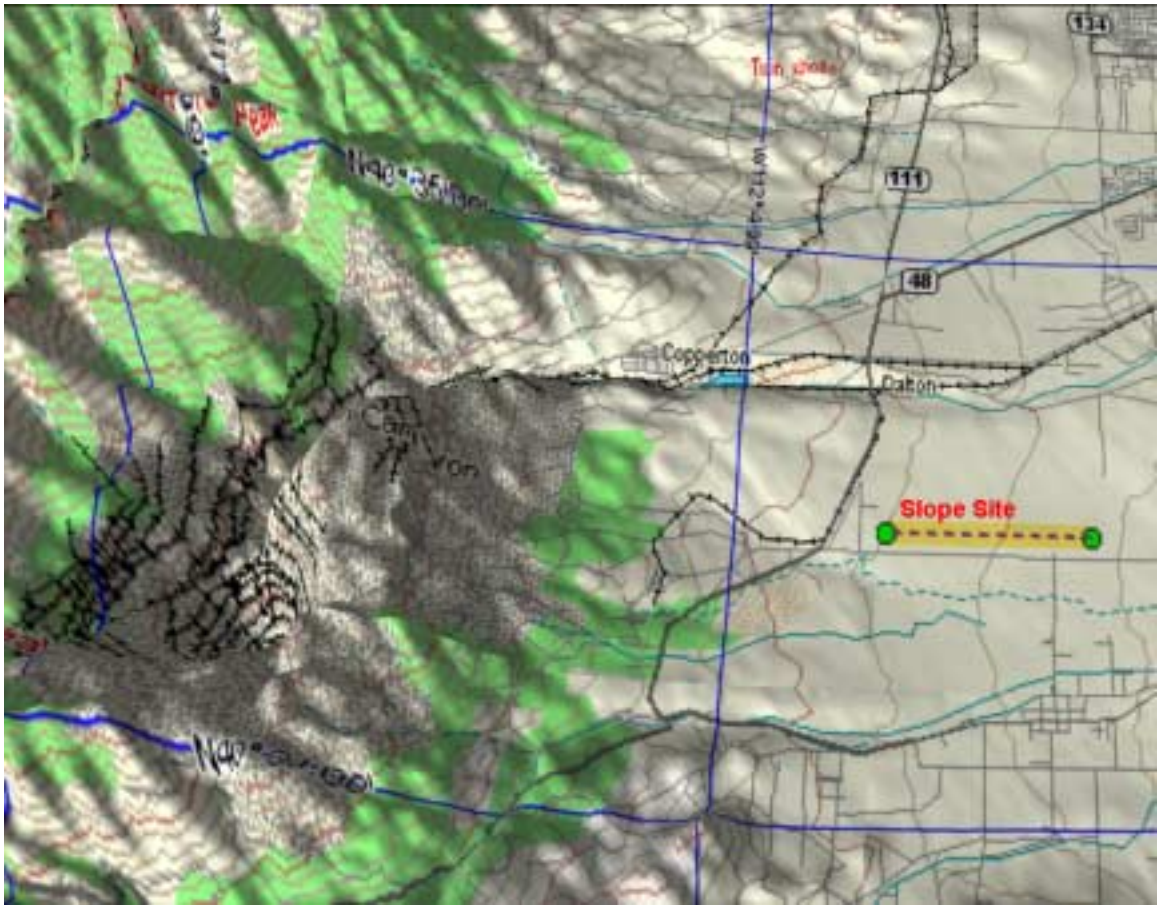
$$\frac{DU}{Dt} \equiv \frac{\partial U}{\partial t} + U\frac{\partial U}{\partial \hat{x}} + W\frac{\partial U}{\partial \hat{z}} \approx -\alpha_H \Delta b + \partial \tau / \partial \hat{z}$$

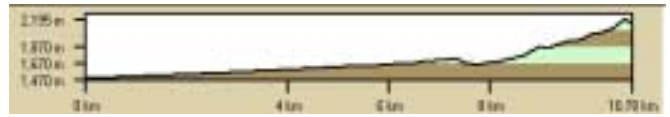
$$\frac{D\Delta b}{Dt} \equiv \frac{\partial \Delta b}{\partial t} + U\frac{\partial \Delta b}{\partial \hat{x}} + W\frac{\partial \Delta b}{\partial \hat{z}} \approx \frac{\partial F_b}{\partial \hat{z}} - \frac{E_b}{h}\Delta b + \alpha_H N_E^2 U_H$$

$$U_H^2 = \lambda_c \alpha_H \Delta b L_H$$

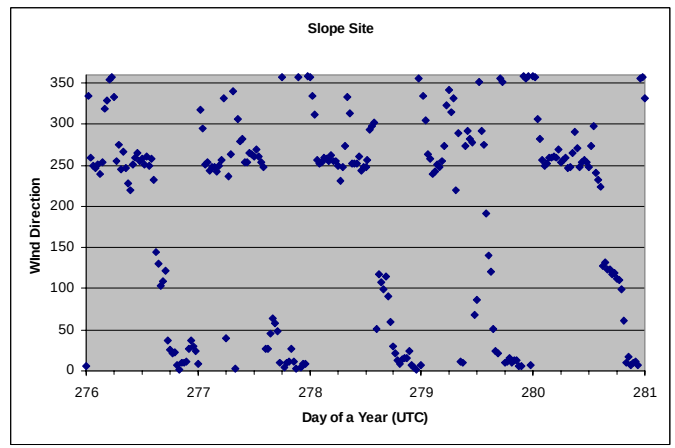


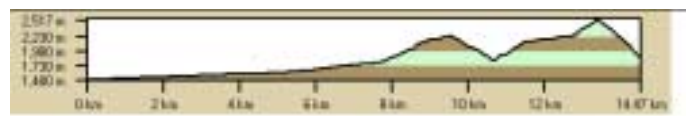
Slope Length – Slope Site





Flow path assumed by Dr. Whiteman, PNNL

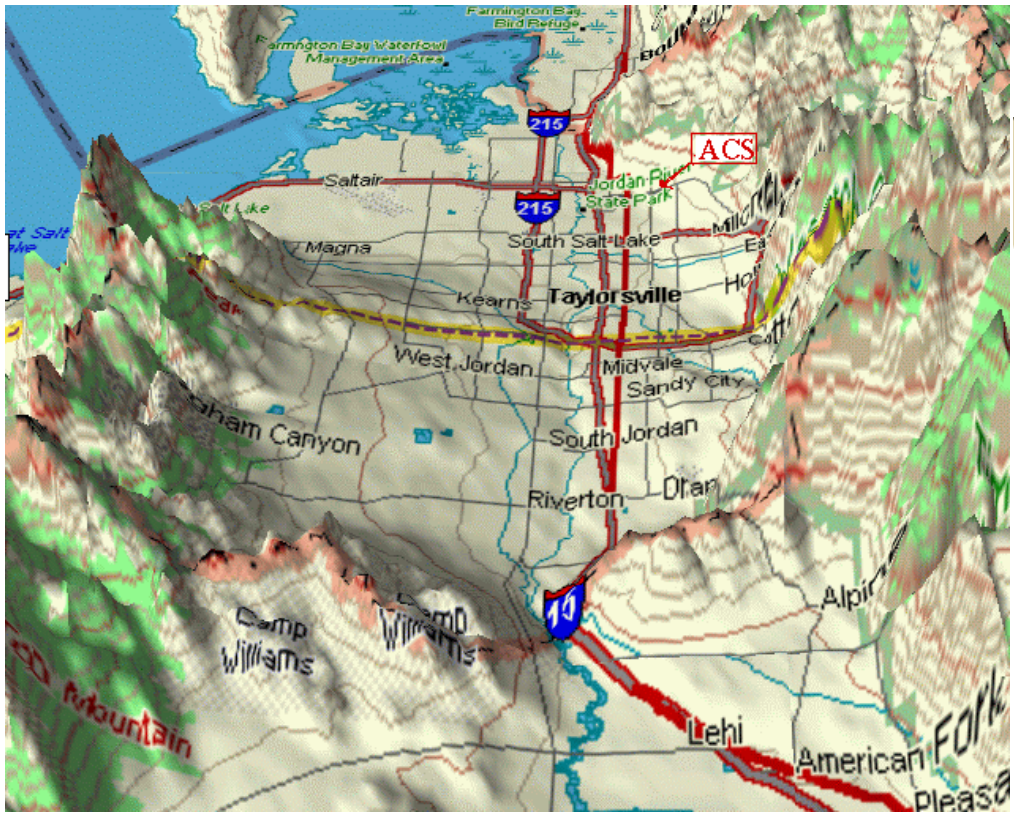


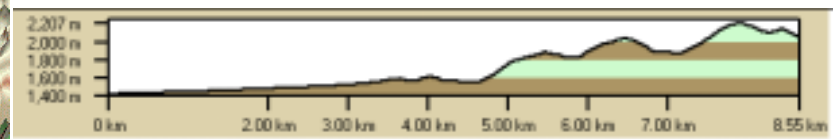


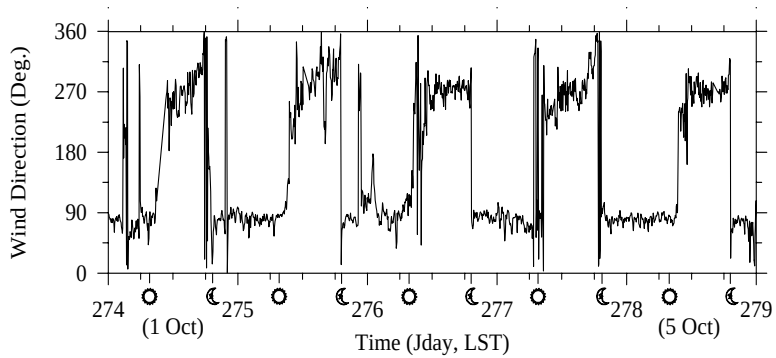
West-East Cross-section

$$U_H^2 = \lambda_c \, \alpha_H \, \Delta b \, L_H$$

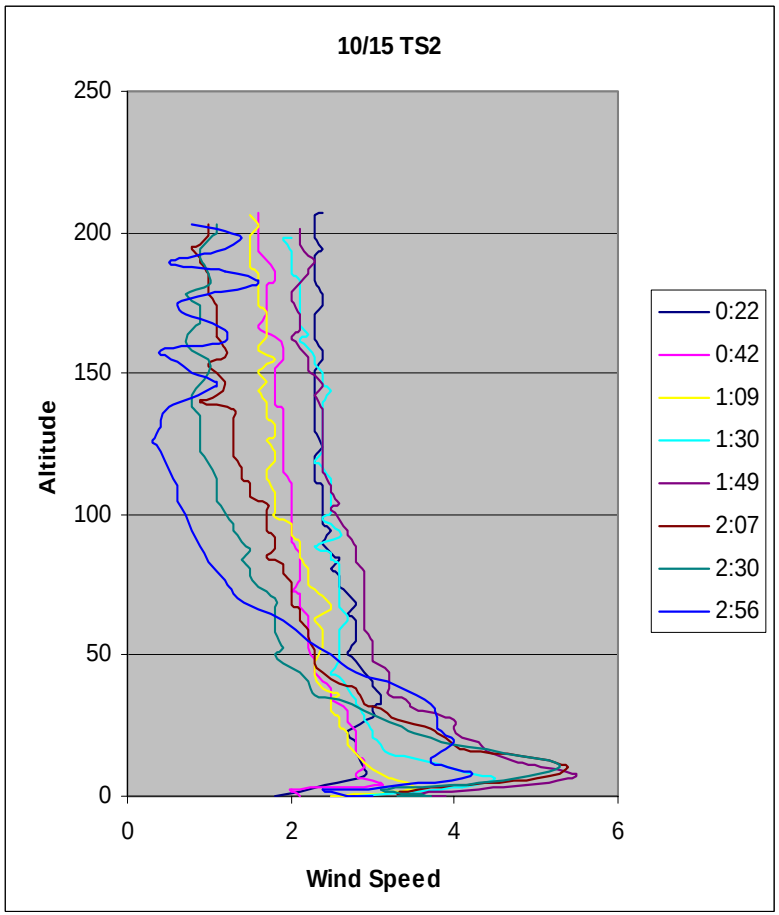
Slope Length – ACS

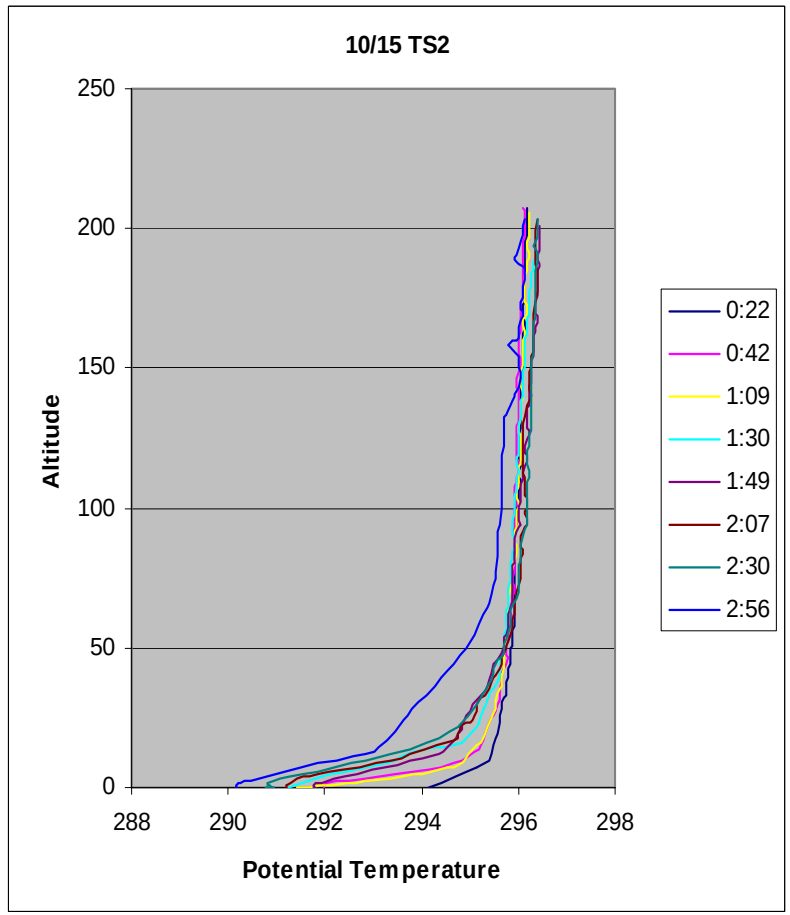






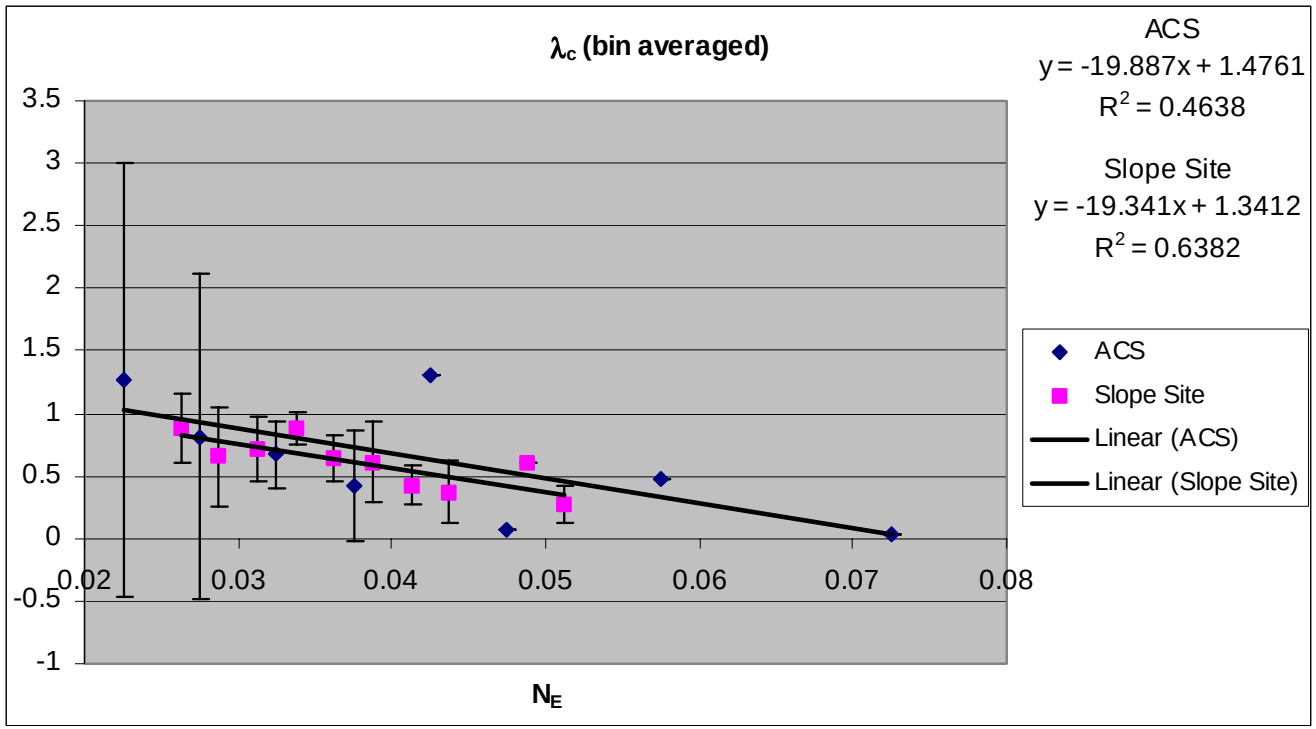
Downslope – Field Data





Downslope – Theory vs. Field

$$U_H^2 = \lambda_c \, \alpha_H \, \Delta b \, L_H$$



Downslope Flow – Theoretical Model – Layer Thickness

Using energy arguments (Turner 1973, Manins & Sawford 1979)

$$\frac{dh_H}{dt} = U_H \cdot \frac{dh_H}{d\hat{x}} \sim \frac{u_{rms}}{(\Delta b\, h_H / u_{rms}^2)}$$

Integrating,

$$h_H^2 \sim \frac{u_{rms}^3 L_H}{U_H \Delta b}$$

Replacing $U_H^2 = \lambda_c \alpha_H \Delta b L_H$,

$$h_H^2 \sim \alpha_H \lambda_C L_H^2 \left(u_{rms} / U_H\right)^3$$

Layer Depth – Nappo and Rao

Katabatic layer depth according to Nappo and Rao (1987):

$$h_I \approx 0.073(\sin\beta)^{2/3} s$$

where β is the slope angle and s the along-slope distance measured downward

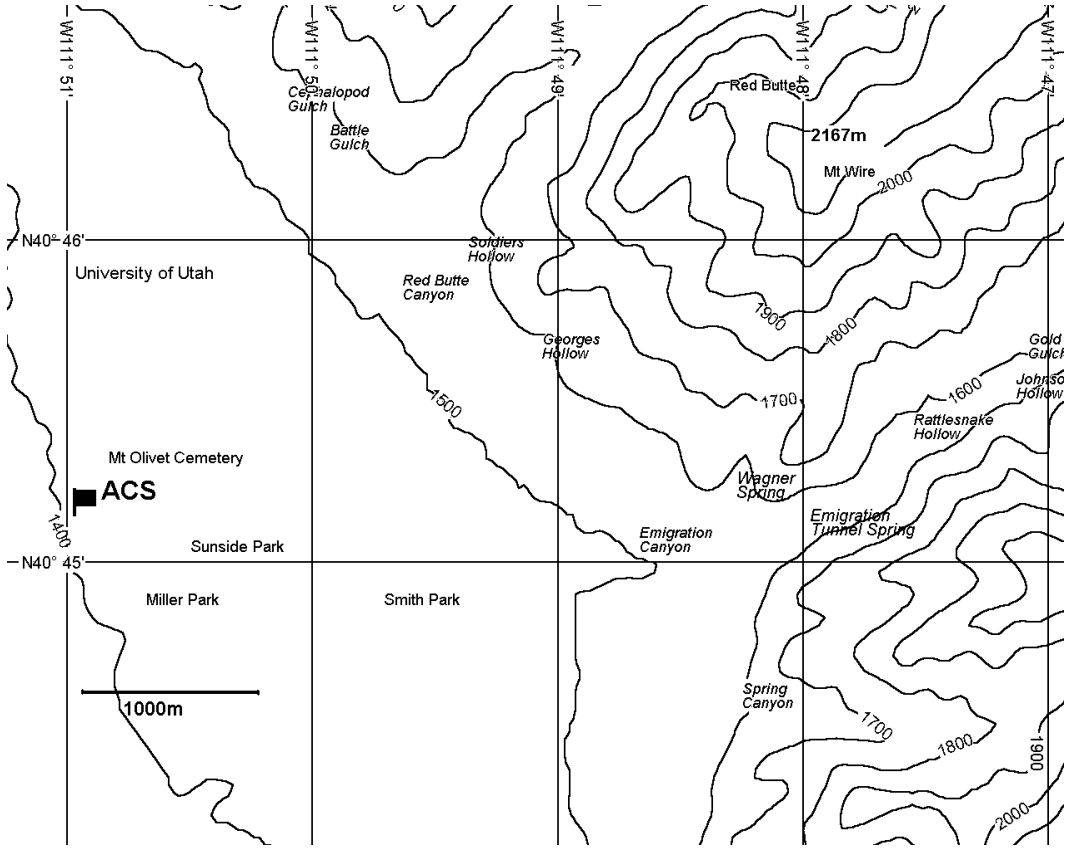
VTMX –Site Surrounding

Site characteristics:

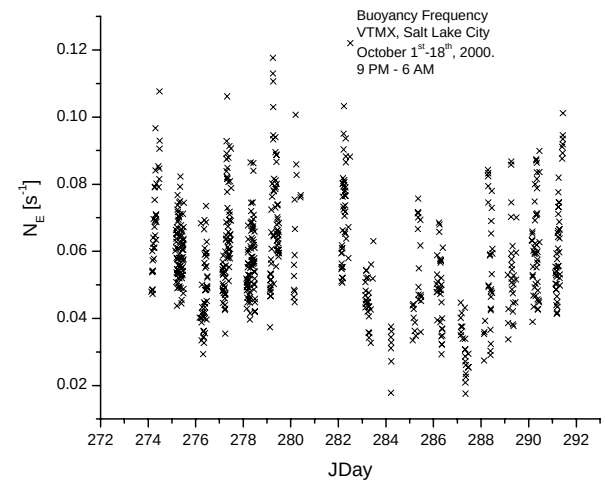
$$\alpha_H = 0.07 rad = 4^o$$

$$L_H = 4000m$$

No direct obstacles for katabatic current.



Stability



Buoyancy frequency,

$$N_E = 0.018 \text{ to } 0.122 s^{-1}$$

Three categories of stability:

Weak $N_E \leq 0.04 s^{-1}$

Moderate $0.04 s^{-1} < N_E \leq 0.08 s^{-1}$

Strong $N_E > 0.08 s^{-1}$

VTMX – Layer thickness

$$h_H^2 \sim \alpha_H \lambda_C L_H^2 \left(u_{rms} / U_H\right)^3$$

Observed katabatic layer thickness is in the range of 15 - 100 m

Whole period $h_H \approx 40m$

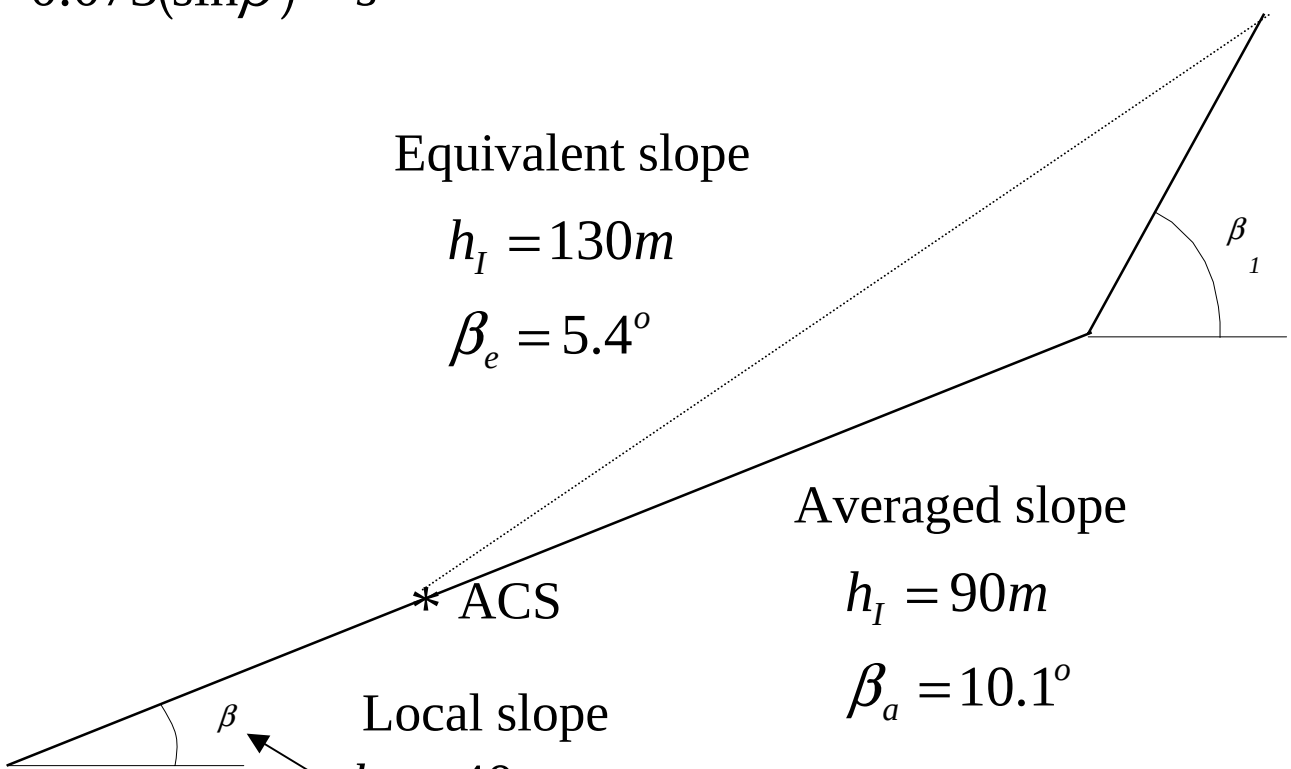
Weak Stability $h_H \approx 70m$

Moderate Stability $h_H \approx 40m$

Strong Stability $h_H \approx 20m$

Layer Depth – Nappo and Rao

$$h_I \approx 0.073(\sin\beta)^{2/3} s$$



Wasatch Mountain

Equivalent slope

$h_I = 130m$

$\beta_e = 5.4^o$

Averaged slope

$h_I = 90m$

$\beta_a = 10.1^o$

Local slope

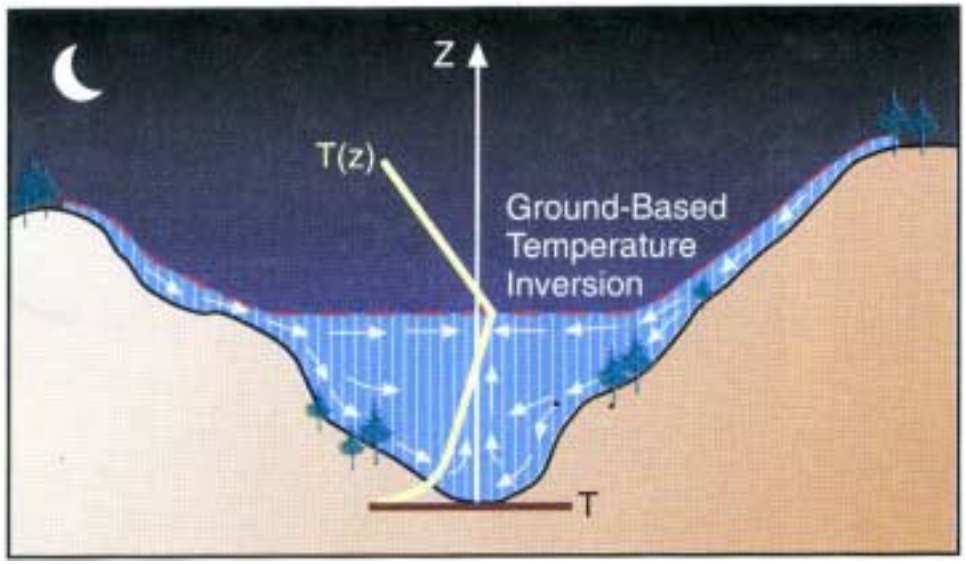
$h_I = 40m$

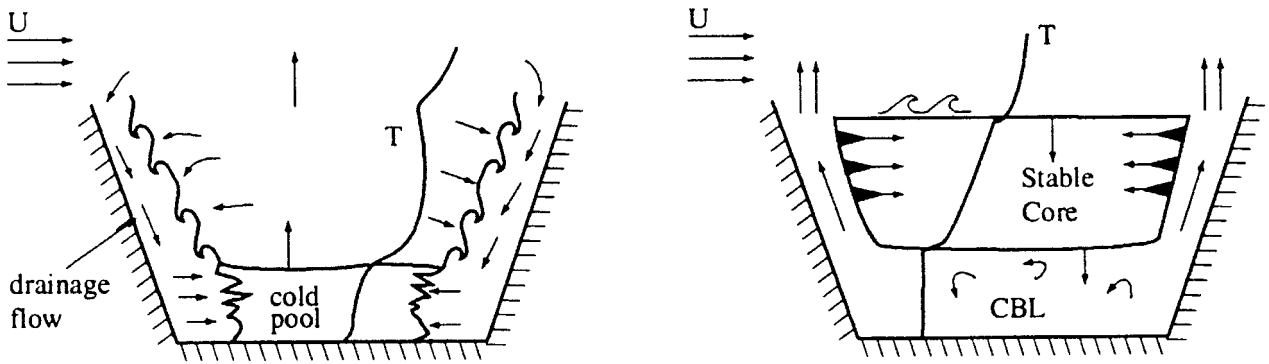
$\beta_l = 4^o$

Observed

$h_I = 15 - 100m$

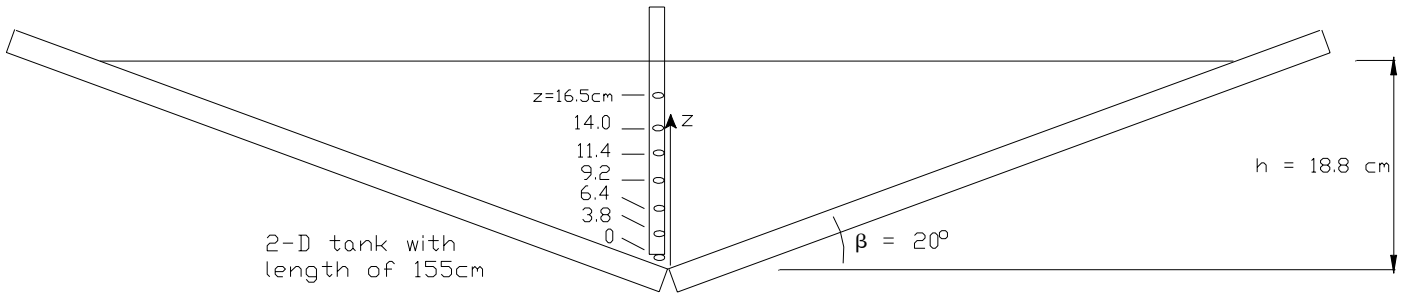
Cold Pool Breakup

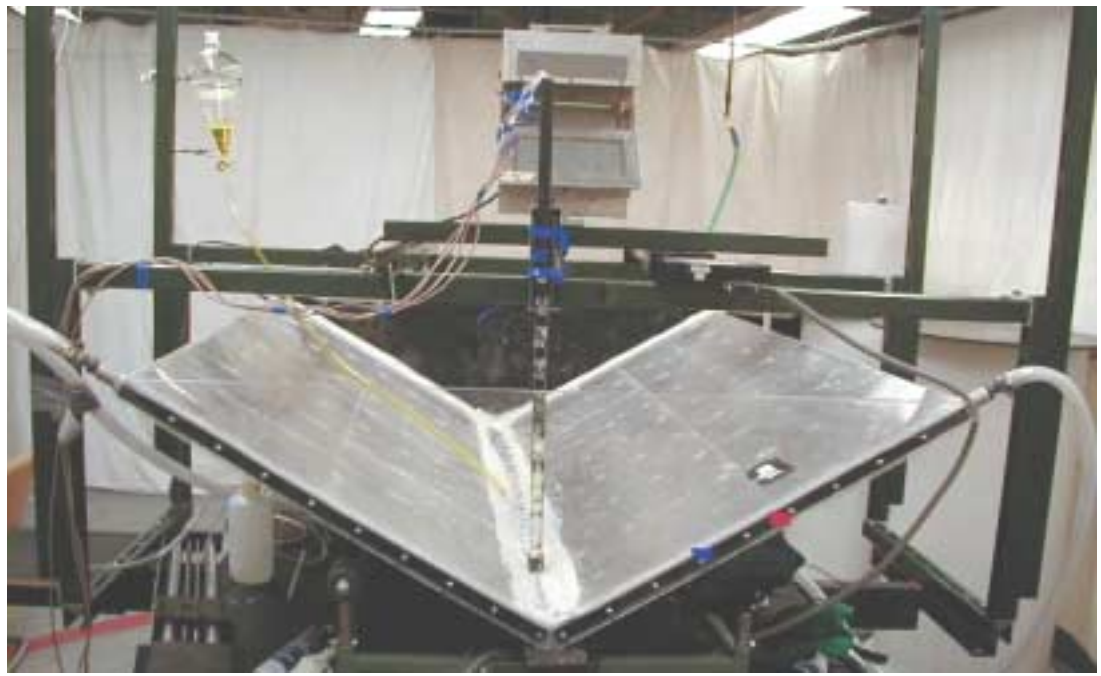




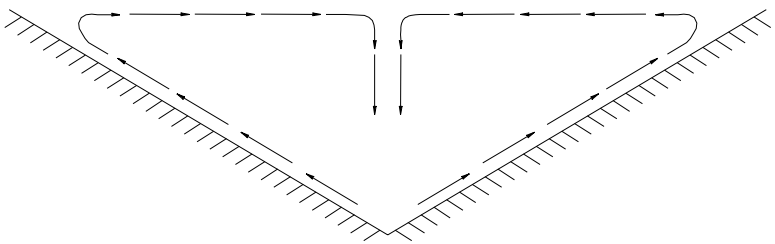
Reproduced from *Mountain Meteorology* (2000), and *Observations of thermally developed wind systems in mountainous terrain* (1990). Courtesy of Dr. Whiteman, PNNL.

Experimental Modeling of the Cold Pool Destruction

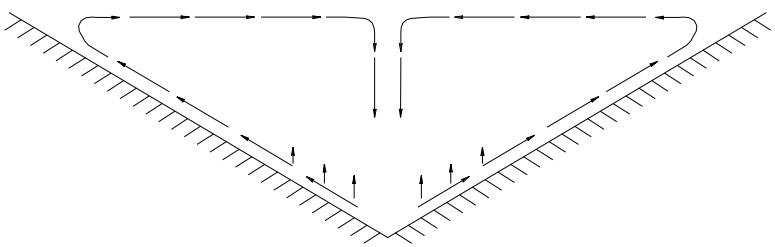




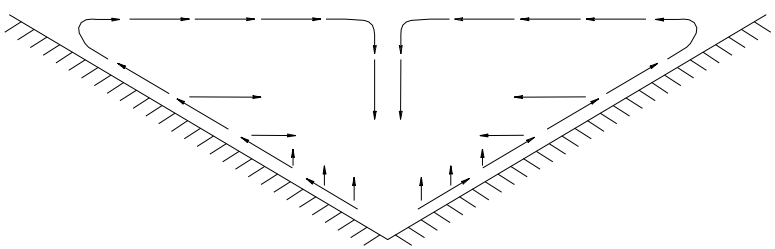
Different Flows Observed During Cold Pool Destruction



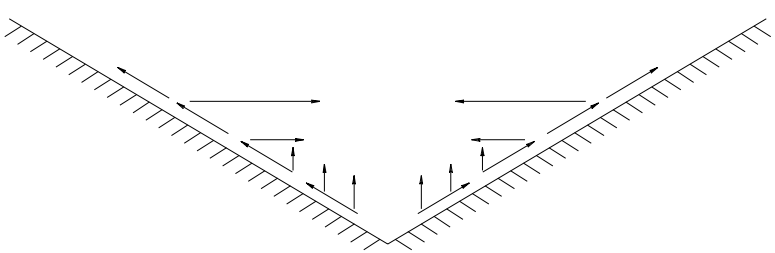
Simple slope flow followed by recirculation



Slope flow followed by recirculation plus layer “thickening” at the valley bottom



Same as previous plus horizontal intrusions in stable core



No large recirculation – all compensation of mass is via intrusions at different levels

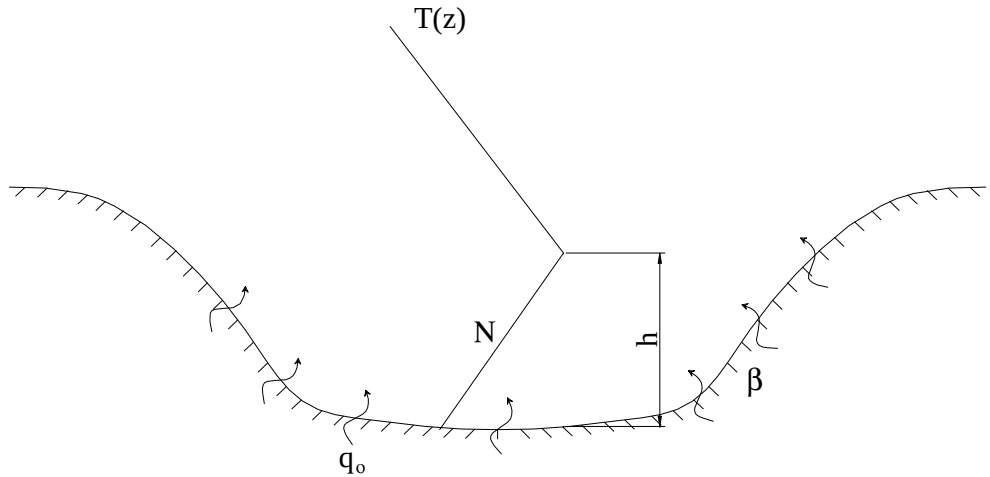
Governing Parameters For the Cold Pool Destruction (2D)

Slope Angle - β

Initial Stability (stratification) - *N*

Heat Flux (buoyancy flux) - q_o

Inversion Height - *h*



$$q_o = \frac{\alpha g Q^*}{\rho c_p}$$

Combination of dimensionless parameters:

β and $B = \dfrac{N^3 h^2}{q_o}$

Parameter B

Assumption: B determines the type of the Cold Pool destruction

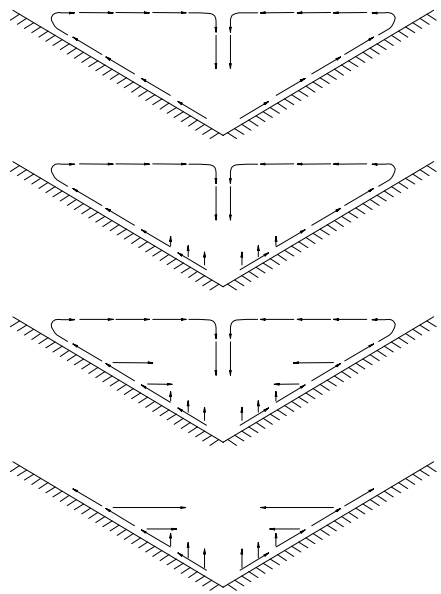
$$B = \frac{N^3 h^2}{q_o}$$

High B → High *N*, Low q_o

High B → Low *N*, Lower q_o

Low B → Low *N*, High q_o

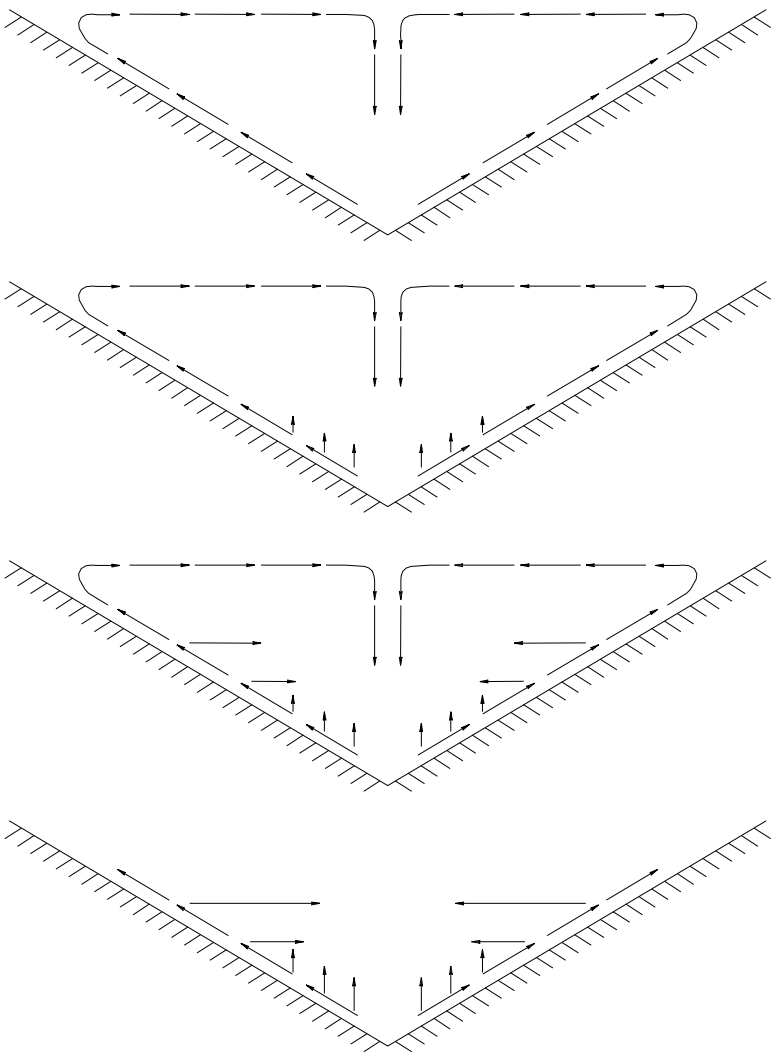
Low B → High *N*, Higher q_o



Run#	N^2 (s^-2)	heatFlux (kW/m^2)	q_0 (m^2/s^3)	Raf	B
35	0.278	1.42	7.21E-07	5.09E+10	2166
36	0.157	0.52	2.66E-07	1.88E+10	2497
40	0.181	5.68	2.89E-06	2.04E+11	943
42	0.287	11.36	5.78E-06	4.08E+11	939

Flow Dependence

Angle	B_{min}	B_{max}
10°	107	2497
20°	212	8198
30°	24	5564



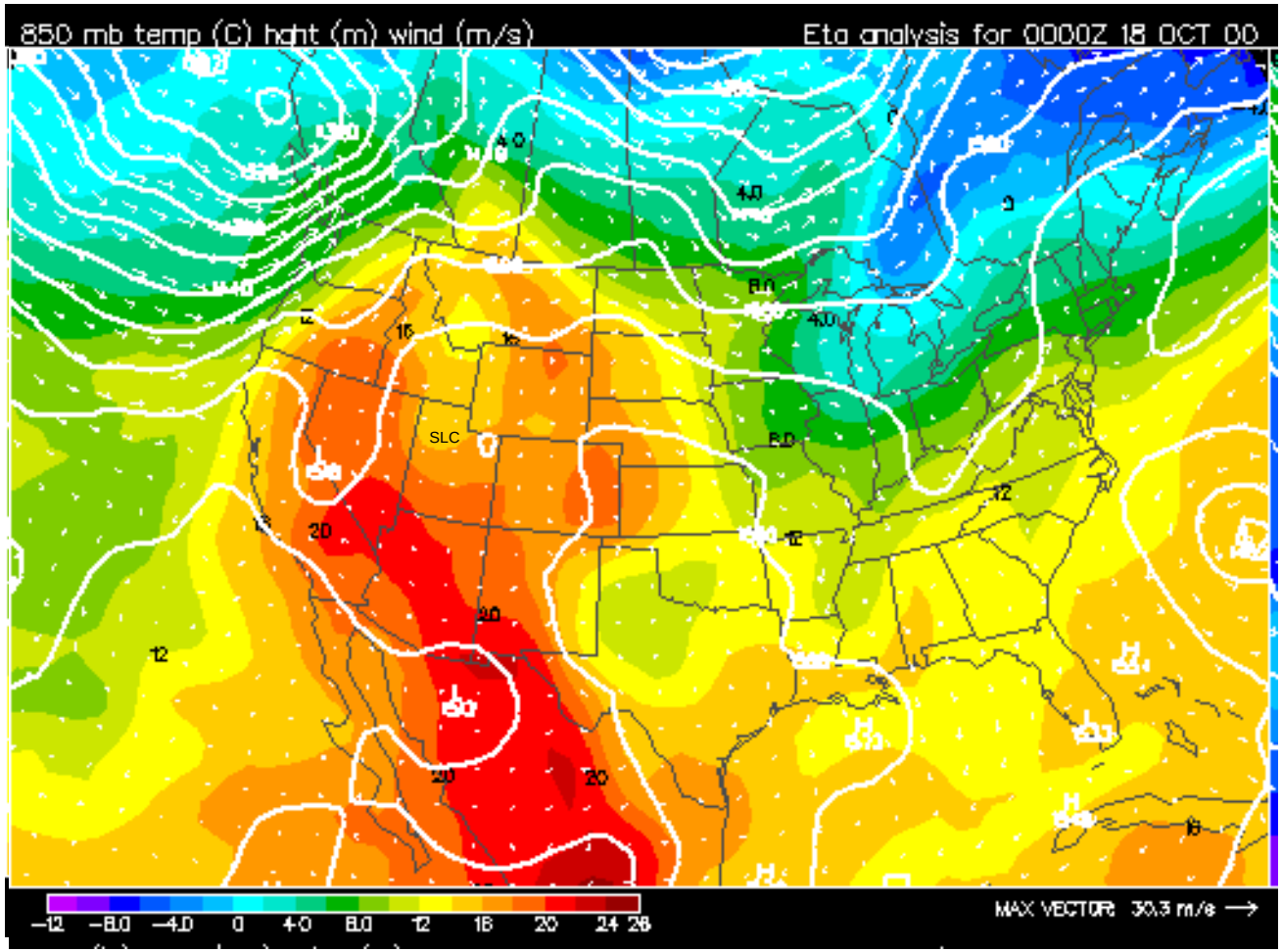
Low B regime

High B regime

B_c=1000-2000

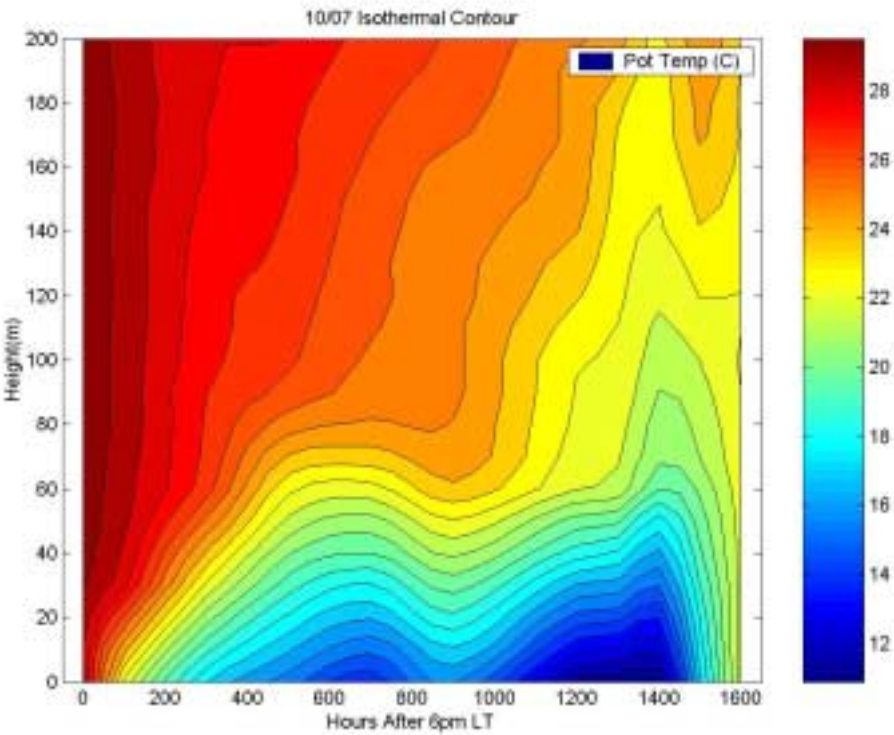
Lower values for smaller slope angles

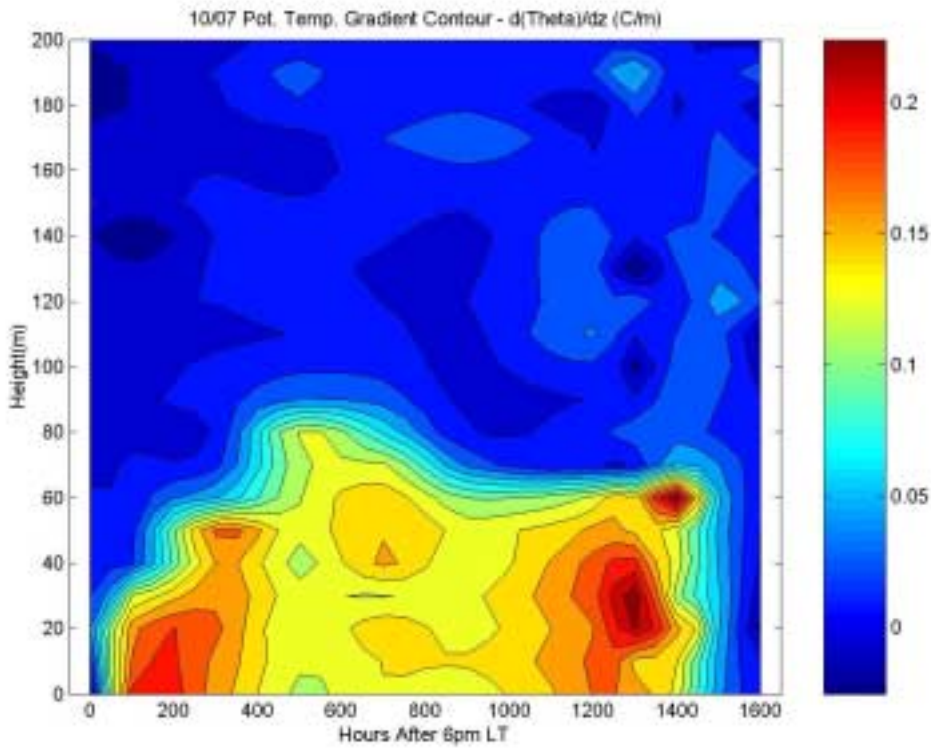
Warm Air Advection



October 17th, 2000. 6 Pm Local Time

Temperature and Gradient Contours





Flow Rates and Pool Growth

Flow in:

9,183,000 – 16,529,000 m^3/s

If everything flows out, necessary velocity towards lake should be :

10.5-19 m/s !

Velocity towards lake ranges from:

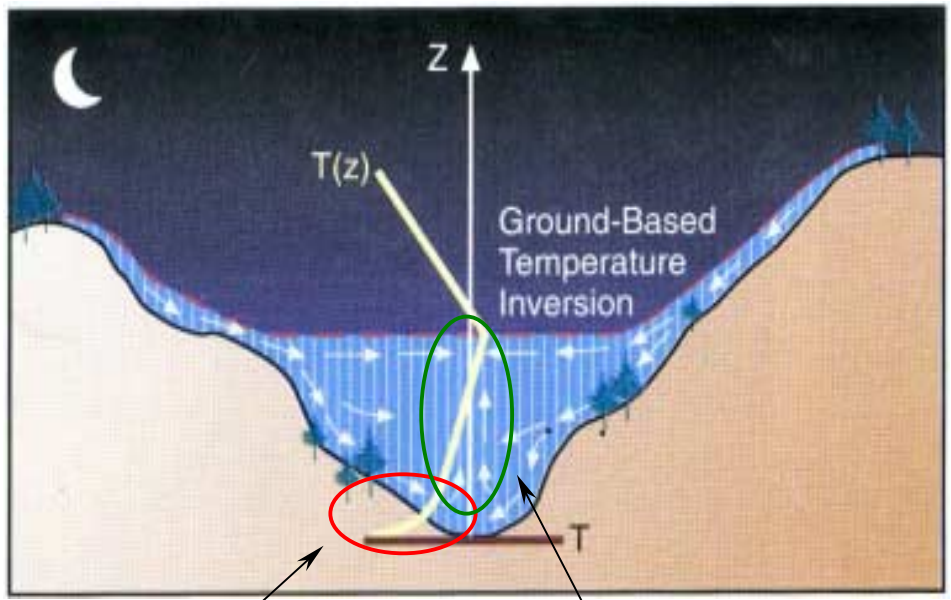
2.5 – 5 m/s

Difference between flow in and flow out creates the cold pool. Rate of pool growth is estimated in the range of:

0.6 – 1.3 cm/s

(100 m increase in depth in 2 – 4.5 hours)

Estimated Pool Height



Very stable, due to the radiative cooling

50 –100 m

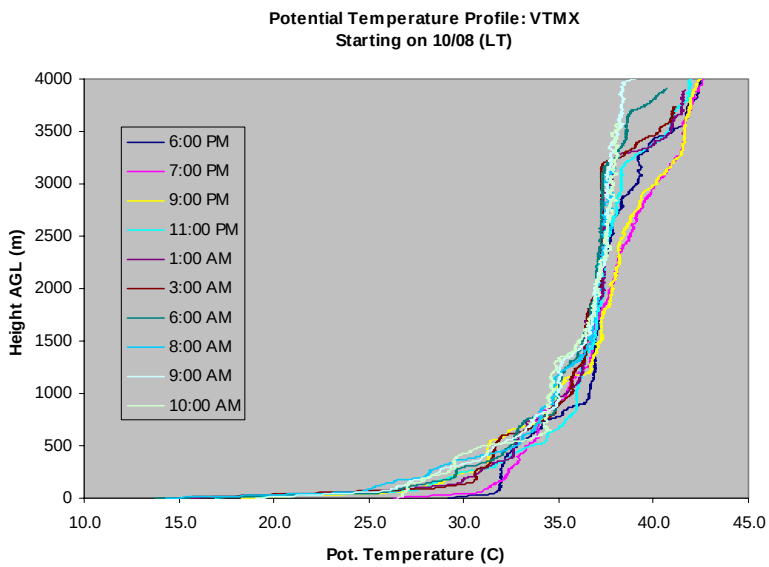
Accumulation of the cold air due to the drainage flow

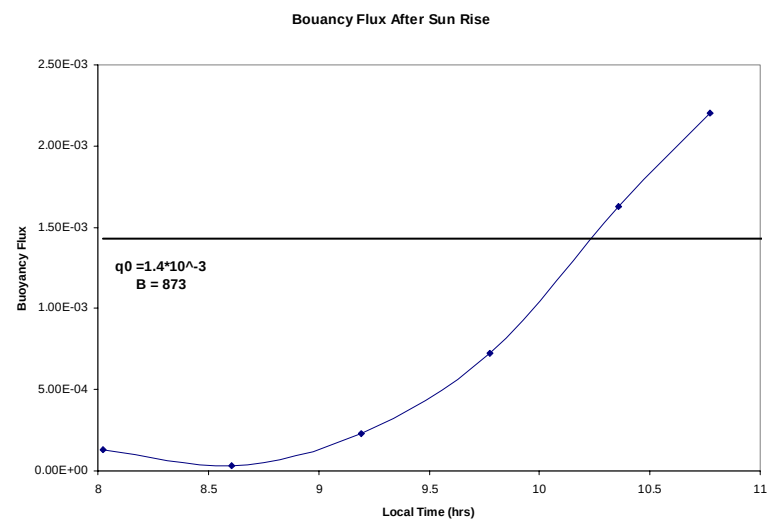
300 – 700 m

Total expected pool depth 350 – 800 m

Expected Cold Pool Destruction Mechanism SLC Valley

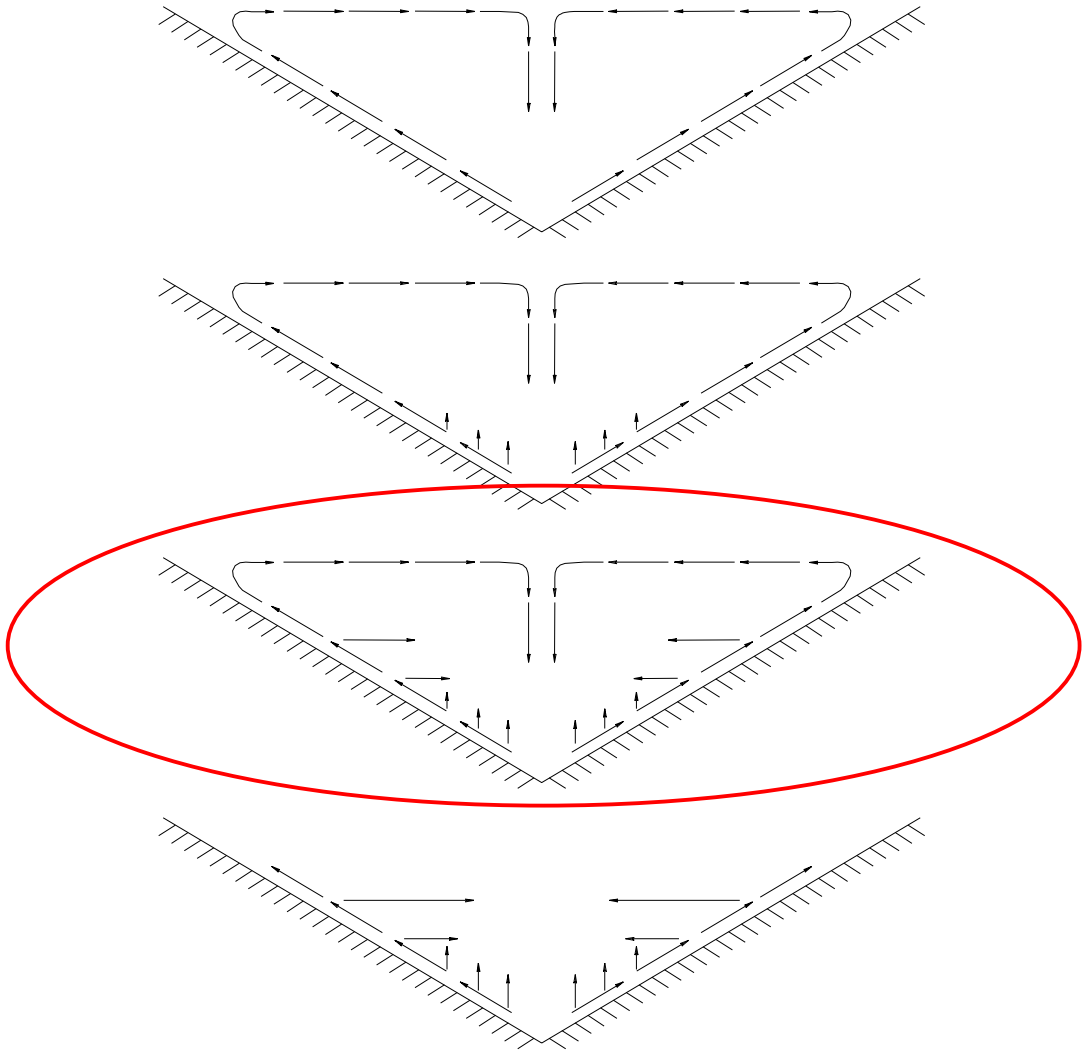
$$B = \frac{N^3 h^2}{q_0}$$





$$B \approx 1000$$

Expected Cold Pool Destruction Mechanism SLC Valley



$B \approx 1000$

Summary

-Dynamics of downslope flows, in particular slope discontinuity were studied

-Existing parameterizations were checked and new were proposed

-Velocity and layer thickness were measured and compared with existing theory / new formulations

-Cold pool formation and destruction were studied using

-Laboratory experiments

-VTMX data

-Pooling mechanisms for VTMX was proposed based on laboratory experiments and simple estimates based on downslope velocities

-Warm advection event was noted

-MM5 runs are underway